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Proteus Two-Dimensional Navier-Stokes Computer Code—Version 2.0

Volume 2—User's Guide

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PRINCIPAL NOTATION

SYMBOLS

Unless specified otherwise, all variables are nondimensional.

<u>Symbol</u>	<u>Definition</u>
c_p, c_v	Specific heats at constant pressure and volume.
c_p	Static pressure coefficient.
E_T	Total energy per unit volume.
g_c	Proportionality constant in Newton's second law.
h_T	Stagnation enthalpy per unit mass.
k	Effective thermal conductivity coefficient.
L_r	Dimensional reference length.
M	Mach number.
n	Time level.
N_1, N_2	Number of grid points in the ξ and η directions.
p	Static pressure.
Pr_l	Laminar Prandtl number.
Q	Vector of dependent variables in the Cartesian or cylindrical coordinate form of the governing equations.
R	Residual.
R	Gas constant.
Re_r	Reference Reynolds number.
T	Static temperature.
u, v	Velocities in the Cartesian x and y directions.
u, v, w	Velocities in the cylindrical x, r , and swirl directions.
x, r	Cylindrical axial and radial coordinates.
x, y	Cartesian coordinates.
γ	Ratio of specific heats, c_p/c_v .
$\varepsilon_E^{(2)}, \varepsilon_E^{(4)}$	Second- and fourth-order explicit artificial viscosity coefficients in constant coefficient model.
ε_I	Implicit artificial viscosity coefficient.
θ	Cylindrical circumferential coordinate.
$\theta_1, \theta_2, \theta_3$	Parameters determining type of time differencing used.
κ_2, κ_4	Constants in nonlinear coefficient artificial viscosity model.
μ	Viscosity coefficient.
ξ, η	Computational coordinate directions.

SymbolDefinition ρ

Static density.

 τ

Computational time.

SUBSCRIPTSSubscriptDefinition i, j Denotes grid location in ξ and η directions. n

Denotes dimensional normalizing condition.

 r

Denotes dimensional reference condition.

 T

Denotes total, or stagnation, value.

SUPERSCRIPTSSuperscriptDefinition n

Denotes time level.

 $\bar{}$

Overbar; denotes dimensional value.

PROTEUS TWO-DIMENSIONAL NAVIER-STOKES COMPUTER CODE - VERSION 2.0

Volume 2 - User's Guide

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SUMMARY

A computer code called *Proteus* has been developed to solve the two-dimensional planar or axisymmetric, Reynolds-averaged, unsteady compressible Navier-Stokes equations in strong conservation law form. The objective in this effort has been to develop a code for aerospace propulsion applications that is easy to use and easy to modify. Code readability, modularity, and documentation have been emphasized.

The governing equations are written in Cartesian coordinates and transformed into generalized nonorthogonal body-fitted coordinates. They are solved by marching in time using a fully-coupled alternating-direction-implicit solution procedure with generalized first- or second-order time differencing. The boundary conditions are also treated implicitly, and may be steady or unsteady. Spatially periodic boundary conditions are also available. All terms, including the diffusion terms, are linearized using second-order Taylor series expansions. Turbulence is modeled using either an algebraic or two-equation eddy viscosity model.

The program contains many operating options. The governing equations may be solved for two-dimensional planar flow, or axisymmetric flow with or without swirl. The thin-layer or Euler equations may be solved as subsets of the Navier-Stokes equations. The energy equation may be eliminated by the assumption of constant total enthalpy. Explicit and implicit artificial viscosity may be used to damp pre- and post-shock oscillations in supersonic flow and to minimize odd-even decoupling caused by central spatial differencing of the convective terms in high Reynolds number flow. Several time step options are available for convergence acceleration, including a locally variable time step and global time step cycling. Simple Cartesian or polar grids may be generated internally by the program. More complex geometries require an externally generated computational coordinate system.

The documentation is divided into three volumes. Volume 1 is the Analysis Description, and presents the equations and solution procedure used in *Proteus*. It describes in detail the governing equations, the turbulence model, the linearization of the equations and boundary conditions, the time and space differencing formulas, the ADI solution procedure, and the artificial viscosity models. Volume 2, the current volume, is the User's Guide, and contains information needed to run the program. It describes the program's general features, the input and output, the procedure for setting up initial conditions, the computer resource requirements, the diagnostic messages that may be generated, the job control language used to run the program, and several test cases. Volume 3 is the Programmer's Reference, and contains detailed information useful when modifying the program. It describes the program structure, the Fortran variables stored in common blocks, and the details of each subprogram.

Version 1.0 of the two-dimensional *Proteus* code was released in late 1989. The current documentation covers Version 2.0, released in early 1992.

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1.0 INTRODUCTION

Much of the effort in applied computational fluid dynamics consists of modifying an existing program for whatever geometries and flow regimes are of current interest to the researcher. Unfortunately, nearly all of the available non-proprietary programs were started as research projects with the emphasis on demonstrating the numerical algorithm rather than ease of use or ease of modification. The developers usually intend to clean up and formally document the program, but the immediate need to extend it to new geometries and flow regimes takes precedence.

The result is often a haphazard collection of poorly written code without any consistent structure. An extensively modified program may not even perform as expected under certain combinations of operating options. Each new user must invest considerable time and effort in attempting to understand the underlying structure of the program if intending to do anything more than run standard test cases with it. The user's subsequent modifications further obscure the program structure and therefore make it even more difficult for others to understand.

The *Proteus* two-dimensional Navier-Stokes computer program is a user-oriented and easily-modifiable flow analysis program for aerospace propulsion applications. Readability, modularity, and documentation were primary objectives during its development. The entire program was specified, designed, and implemented in a controlled, systematic manner. Strict programming standards were enforced by immediate peer review of code modules; Kernighan and Plauger (1978) provided many useful ideas about consistent programming style. Every subroutine contains an extensive comment section describing the purpose, input variables, output variables, and calling sequence of the subroutine. With just three clearly-defined exceptions, the entire program is written in ANSI standard Fortran 77 to enhance portability. A master version of the program is maintained and periodically updated with corrections, as well as extensions of general interest (e.g., turbulence models.)

The *Proteus* program solves the unsteady, compressible, Reynolds-averaged Navier-Stokes equations in strong conservation law form. The governing equations are written in Cartesian coordinates and transformed into generalized nonorthogonal body-fitted coordinates. They are solved by marching in time using a fully-coupled alternating-direction-implicit (ADI) scheme with generalized time and space differencing (Briley and McDonald, 1977; Beam and Warming, 1978). Turbulence is modeled using either the Baldwin and Lomax (1978) algebraic eddy-viscosity model or the Chien (1982) two-equation model. All terms, including the diffusion terms, are linearized using second-order Taylor series expansions. The boundary conditions are treated implicitly, and may be steady or unsteady. Spatially periodic boundary conditions are also available.

The program contains many operating options. The governing equations may be solved for two-dimensional planar flow, or axisymmetric flow with or without swirl. The thin-layer or Euler equations may be solved as subsets of the Navier-Stokes equations. The energy equation may be eliminated by the assumption of constant total enthalpy. Explicit and implicit artificial viscosity may be used to damp pre- and post-shock oscillations in supersonic flow and to minimize odd-even decoupling caused by central spatial differencing of the convective terms in high Reynolds number flow. Several time step options are available for convergence acceleration, including a locally variable time step and global time step cycling. Simple grids may be generated internally by the program; more complex geometries require external grid generation, such as that developed by Chen and Schwab (1988).

The documentation is divided into three volumes. Volume 1 is the Analysis Description, and presents the equations and solution procedure used in *Proteus*. It describes in detail the governing equations, the turbulence model, the linearization of the equations and boundary conditions, the time and space differencing formulas, the ADI solution procedure, and the artificial viscosity models. Volume 2, the current volume, is the User's Guide, and contains information needed to run the program. It describes the program's general features, the input and output, the procedure for setting up initial conditions, the computer

resource requirements, the diagnostic messages that may be generated, the job control language used to run the program, and several test cases. Volume 3 is the Programmer's Reference, and contains detailed information useful when modifying the program. It describes the program structure, the Fortran variables stored in common blocks, and the details of each subprogram.

Version 1.0 of the two-dimensional *Proteus* code was released in late 1989 (Towne, Schwab, Benson, and Suresh, 1990). The current documentation covers Version 2.0, released in early 1992.

The authors would like to acknowledge the significant contributions made by their co-workers. Tom Benson provided part of the original impetus for the development of *Proteus*, and did the original coding of the block tri-diagonal inversion routines. Simon Chen did the original coding of the Baldwin-Lomax turbulence model, and consulted in the implementation of the nonlinear coefficient artificial viscosity model. William Kunik developed the original code for computing the metrics of the generalized nonorthogonal grid transformation. Frank Molls has created separate diagonalized and patched-grid versions of the code. Ambady Suresh did the original coding for the second-order time differencing and for the nonlinear coefficient artificial viscosity model. These people, along with Dick Cavicchi, Julie Conley, Jason Solbeck, and Pat Zeman, have also run many debugging and verification cases.

2.0 GENERAL DESCRIPTION

In this section the basic characteristics and capabilities of the *Proteus* code are described in general. More detailed descriptions can be found in other sections of this manual or in Volumes 1 and 3.

2.1 ANALYSIS

Proteus 2-D solves the two-dimensional planar or axisymmetric unsteady compressible Navier-Stokes equations. Swirl is allowed in axisymmetric flow. The planar equations are solved in fully conservative form. As subsets of these equations, options are available to solve the Euler equations or the thin-layer Navier-Stokes equations. An option is also available to eliminate the energy equation by assuming constant total enthalpy. The governing equations are described in detail in Section 2.0 of Volume 1.

The equations are solved by marching in time using the generalized time differencing of Beam and Warming (1978). The method may be either first- or second-order accurate in time, depending on the choice of time differencing parameters. Second-order central differencing is used for all spatial derivatives. The time and space differencing formulas are presented in Sections 3.0 and 5.0 of Volume 1. Nonlinear terms are linearized using second-order Taylor series expansions in time, as described in Section 4.0 of Volume 1. The resulting difference equations are solved using an alternating-direction implicit (ADI) technique, with Douglas-Gunn type splitting as written by Briley and McDonald (1977). The boundary conditions are also treated implicitly.

Artificial viscosity, or smoothing, is normally added to the solution algorithm to damp pre- and post-shock oscillations in supersonic flow, and to prevent odd-even decoupling due to the use of central differences in convection-dominated regions of the flow. Implicit smoothing and two types of explicit smoothing are available in *Proteus*. The implicit smoothing is second order with constant coefficients. For the explicit smoothing the user may choose a constant coefficient second- and/or fourth-order model (Steger, 1978), or a nonlinear coefficient mixed second- and fourth-order model (Jameson, Schmidt, and Turkel, 1981). The nonlinear coefficient model was designed specifically for flow with shock waves. The artificial viscosity models are described in detail in Section 8.0 of Volume 1.

The equations are fully coupled, leading to a system of equations with a block tridiagonal coefficient matrix that can be solved using the block matrix version of the Thomas algorithm. Because this algorithm is recursive, the source code cannot be vectorized in the ADI sweep direction. However, it is vectorized in the non-sweep direction, leading to an efficient implementation of the algorithm. The solution algorithm is described in detail in Section 7.0 of Volume 1.

2.2 GEOMETRY AND GRID SYSTEM

The equations solved in *Proteus* were originally written in a Cartesian coordinate system, then transformed into a general nonorthogonal computational coordinate system as described in Section 2.3 of Volume 1. The code is therefore not limited to any particular type of geometry or coordinate system. The only requirement is that body-fitted coordinates must be used. In general, the computational coordinate system for a particular geometry must be created by a separate coordinate generation code and stored in an unformatted file that *Proteus* can read. However, simple Cartesian and polar coordinate systems are built in.

The equations are solved at grid points that form a computational mesh within this computational coordinate system. Note that a distinction is being made between the terms *computational coordinate system* and *computational mesh*. The *computational coordinate system* refers to the (ξ, η) system in which the governing equations are written. It is determined by supplying a series of points whose Cartesian (x, y) coordinates are specified, either by reading them from a file or through one of the analytically defined coordinate systems built into subroutine GEOM. The *computational mesh* consists of grid points distributed along lines in the computational coordinate directions. These points may differ in number and location from

those used to determine the computational coordinate system. The number of grid points in each direction in the computational mesh is specified by the user. The location of these grid points can be varied by packing them at either or both boundaries in any coordinate direction. The transformation metrics and Jacobian are computed using finite differences in a manner consistent with the differencing of the governing equations.

2.3 FLOW AND REFERENCE CONDITIONS

As stated earlier, the equations solved by *Proteus* are for compressible flow. Incompressible conditions can be simulated by running at a Mach number of around 0.1. Lower Mach numbers may lead to numerical problems. The flow can be laminar or turbulent. The gas constant R is specified by the user, with the value for air as the default. The specific heats c_p and c_v , the molecular viscosity μ , and the thermal conductivity k can be treated as constants or as functions of temperature. The empirical formulas used to relate these properties to temperature are contained in subroutine FTEMP, and can easily be modified if necessary. The perfect gas equation of state is used to relate pressure, density, and temperature. This equation is contained in subroutine EQSTAT, which could also be easily modified if necessary. All equations and variables in the program are nondimensionalized by normalizing values derived from reference conditions specified by the user, with values for sea level air as the default.

2.4 BOUNDARY CONDITIONS

The easiest way to specify boundary conditions in *Proteus* is by specifying the type of boundary (i.e., no-slip adiabatic wall, subsonic inflow, periodic, etc.). The program will then select an appropriate set of conditions for that boundary. For most applications this method should be sufficient. If necessary, however, the user may instead set the individual boundary conditions on any or all of the four computational boundaries.

A variety of individual boundary conditions are built into the *Proteus* code, including: (1) specified values and/or gradients of Cartesian velocities u , v , and w , normal and tangential velocities V_n and V_t , pressure p , temperature T , and density ρ ; (2) specified values of total pressure p_T , total temperature T_T , and flow angle; and (3) linear extrapolation. Another useful boundary condition is a "no change from initial condition" option for u , v , w , p , T , ρ , p_T , and/or T_T . Provision is also made for user-written boundary conditions using subroutines BCF and BCFLIN. Specified gradient boundary conditions may be in the direction of the coordinate line intersecting the boundary or normal to the boundary, and may be computed using two-point or three-point difference formulas. For all of these conditions, the same type and value may be applied over the entire boundary surface, or a point-by-point distribution may be specified. Unsteady and time-periodic boundary conditions are allowed when applied over the entire boundary. The boundary conditions available in *Proteus* are described in detail in Section 3.1.7.

2.5 INITIAL CONDITIONS

Initial conditions are required throughout the flow field to start the time marching procedure. For unsteady flows they should represent a real flow field. A converged steady-state solution from a previous run would be a good choice. For steady flows, the ideal initial conditions would represent a real flow field that is close to the expected final solution.

The initial conditions are set up in subroutine INIT. A default version of INIT is built into *Proteus* that specifies uniform flow with constant flow properties everywhere in the flow field. These conditions, of course, do represent a solution to the governing equations, and for many problems may help minimize starting transients in the time marching procedure. However, realistic initial conditions that are closer to the expected final solution should lead to quicker convergence.

The best choice for initial conditions, therefore, will vary from problem to problem. For this reason *Proteus* does not include a more generalized routine for setting up initial conditions. Instead, the user should supply his or her own version of subroutine INIT to set up the initial conditions for the particular flow being computed. Details on the Fortran variables to be specified by INIT may be found in Section 5.1.

2.6 TIME STEP SELECTION

Several different options are available for choosing the time step $\Delta\tau$, and for modifying it as the solution proceeds. $\Delta\tau$ may be specified directly, or through a value of the Courant-Friedrichs-Lewy (CFL) number. When specifying a CFL number, the time step $\Delta\tau$ may be either *global* (i.e., constant in space) based on the minimum CFL limit, or *local* (i.e., varying in space) based on the local CFL limit. For unsteady time-accurate flows global values should be used, but for steady flows using local values may lead to faster convergence. Options are available to increase or decrease $\Delta\tau$ as the solution proceeds based on the change in the dependent variables. An option is also available to cycle $\Delta\tau$ between two values in a logarithmic progression over a specified number of time steps. The various time step options are described in detail in Section 3.1.9.

2.7 CONVERGENCE

Five options are currently available for determining convergence. The user specifies a convergence criterion ϵ for each of the governing equations. Then, depending on the option chosen, convergence is based on: (1) the absolute value of the maximum change in the conservation variables ΔQ_{max} over a single time step; (2) the absolute value of the maximum change ΔQ_{max} averaged over a specified number of time steps; (3) the L_2 norm of the residual for each equation; (4) the average residual for each equation; or (5) the maximum residual for each equation. These criteria are defined in Section 4.1.6.

2.8 INPUT/OUTPUT

Input to *Proteus* is through a series of namelists¹ and, in general, an unformatted file containing the computational coordinate system. All of the input parameters have default values and only need to be specified by the user if a different value is desired. Reference conditions may be specified in either English or SI units. The namelist and coordinate system input are described in Section 3.0. A restart option is also available, in which the computational mesh and the initial flow field are read from unformatted restart files created during an earlier run. The use of the restart option is described in Sections 3.1.3 and 5.3.

The standard printed output available in *Proteus* includes an echo of the input, boundary conditions, normalizing and reference conditions, the computed flow field, and convergence information. The user controls exactly which flow field parameters are printed, and at which time levels and grid points. Several debug options are also available for detailed printout in various parts of the program. The printed output is described in Section 4.1.

In addition to the printed output, several unformatted files can be written for various purposes. The first is an auxiliary file used for post-processing, usually called a plot file, that can be written at convergence or after the last time step if the solution does not converge. Plot files can be written for the NASA Lewis plotting program CONTOUR or the NASA Ames plotting program PLOT3D. If PLOT3D is to be used, two unformatted files are created, an XYZ file containing the computational mesh and a Q file containing the computed flow field. The plot files are described in detail in Section 4.2. Another unformatted file written by *Proteus* contains detailed convergence information. This file is automatically incremented each time the solution is checked for convergence, and is used to generate the convergence history printout and with Lewis-developed post-processing plotting routines. The contents of the convergence history file are presented in Section 4.3. And finally, two unformatted files may be written at the end of a calculation that may be used to restart the calculation in a later run. One of these contains the computational mesh, and the other the computed flow field. The contents of the restart files are described in detail in Section 4.4.

2.9 TURBULENCE MODELS

For turbulent flow, *Proteus* solves the Reynolds time-averaged Navier-Stokes equations, with turbulence modeled using either the Baldwin and Lomax (1978) algebraic eddy-viscosity model or the Chien (1982) two-equation model. Both models are described in detail in Section 9.0 of Volume 1.

¹ It should be noted that namelist input is not part of ANSI standard Fortran 77, but is nevertheless available with most Fortran compilers. See Section 2.3.1 of Volume 3 for a discussion of possible computer-dependent features in the *Proteus* code.

2.9.1 Baldwin-Lomax Model

The Baldwin-Lomax model may be applied to either wall-bounded flows or to free turbulent flows. For wall-bounded flows, the model is a two-layer model. In the inner region, in addition to the Baldwin-Lomax formulation, an alternate expression first presented by Spalding (1961), and later by Kleinstein (1967), may be used. For flows in which more than one boundary is a solid surface, averaging procedures are used to determine a single μ_t profile. A simple transition model proposed by Cebeci and Bradshaw (1984) may be used to simulate laminar/turbulent transition. The turbulent thermal conductivity coefficient k_t is computed using Reynolds analogy.

2.9.2 Chien k - ϵ Model

With the Chien two-equation model, partial differential equations are solved for the turbulent kinetic energy k and the turbulent dissipation rate ϵ . These equations are lagged in time and solved separately from the Navier-Stokes equations. The equations are solved using the same solution algorithm as for the mean flow equations, except that spatial derivatives for the convective terms are approximated using first-order upwind differencing.

Since the Chien two-equation model is a low Reynolds number formulation, the k - ϵ equations are solved in the near-wall region. No additional approximations are needed. Boundary conditions that may be used include: (1) no change from initial or restart conditions for k and ϵ ; (2) specified values and/or gradients of k and ϵ ; and (3) linear extrapolation. Specified gradient boundary conditions are in the direction of the coordinate line intersecting the boundary, and may be computed using two-point or three-point difference formulas. For all of these conditions, the same type and value may be applied over the entire boundary surface, or a point-by-point distribution may be specified. Spatially periodic boundary conditions for k and ϵ may also be used. Unsteady boundary conditions are not available for the k - ϵ equations. However, unsteady flows can still be computed with the k - ϵ model using the unsteady boundary condition option for the mean flow quantities and appropriate boundary conditions for k and ϵ , such as specified gradients or linear extrapolation.

Initial conditions for k and ϵ are required throughout the flow field to start the time marching procedure. The best choice for initial conditions for the k - ϵ equations will vary from problem to problem, and the user may supply a subroutine, called KEINIT, that sets up the initial values for k and ϵ for the time marching procedure. Details on the Fortran variables to be specified by KEINIT may be found in Section 5.1. A version of KEINIT is built into *Proteus* that computes the initial k and ϵ values from a mean initial or restart flow field based on the assumption of local equilibrium (i.e., production equals dissipation.) Variations of that scheme have been found to be useful in computing the k - ϵ initial conditions for a variety of turbulent flows.

The time step used in the solution of the k - ϵ equations is normally the same as the time step used for the mean flow equations. However, the user can alter the time step for the k - ϵ equations, making it larger or smaller than the time step for the mean flow equations, by specifying a multiplication factor. The user can also specify the number of k - ϵ iterations per mean flow iteration.

3.0 INPUT DESCRIPTION

The standard input to the two-dimensional version of *Proteus* consists of a title line and several namelists. Additional input may be provided in the form of a pre-stored unformatted file containing the computational coordinate system. The calculation can also be started by reading the computational mesh and the initial flow field from restart files written during a previous run. This section describes only the standard input and the coordinate system file. The restart file contents and format are described in Section 4.4.

3.1 STANDARD INPUT

All of the standard input parameters have default values and do not need to be specified by the user unless some other value is desired. The type (real or integer) of the input parameters follows standard Fortran convention, unless stated otherwise (i.e., those starting with I, J, K, L, M, or N are integer, and the remainder are real.)² Note that in most, if not all, implementations of Fortran, namelist names and input start in character position 2 or higher in the input line. All of the input, except for namelist IC, is read in subroutine INPUT. Namelist IC is read in subroutine INIT.

3.1.1 Reference and Normalizing Conditions

Unless specified otherwise, all of the input parameters are specified in nondimensional form, with the appropriate *reference* condition as the nondimensionalizing factor. A few words explaining what we mean by *reference* conditions and *normalizing* conditions, and the differences between them, may be helpful at this point.

The *normalizing* conditions are, by definition, the conditions used in nondimensionalizing the governing equations, and are denoted by an *n* subscript. (See Section 2.0 of Volume 1.) These normalizing conditions are defined by six basic *reference* conditions, for length, velocity, temperature, density, viscosity, and thermal conductivity, which are specified by the user. Reference conditions are denoted by an *r* subscript. The normalizing conditions used in *Proteus* are listed in Table 3-1.

Note that for some variables, like pressure, the normalizing condition is dictated by the form of the governing equations once the six basic reference conditions are chosen. Unfortunately, some of these may not be physically meaningful or convenient for use in setting up input conditions. Therefore, some additional reference conditions are defined from the six user-supplied ones. The reference conditions are listed in Table 3-2.

To summarize, the *normalizing* conditions are used to nondimensionalize the governing equations. The average user need not be too concerned about these. The *reference* conditions are the ones used for nondimensionalization of all user-specified input and output parameters.³

² Throughout this User's Guide, elements of the Fortran language, such as input variables and subprogram names, are printed in the text using uppercase letters. However, in most implementations, Fortran is case-insensitive. The *Proteus* source code itself is written in lowercase.

³ Internal to the *Proteus* computer code itself, variables are generally nondimensionalized by the normalizing conditions. The reference conditions are used for input and output because they are usually more physically meaningful for the user.

3.1.2 Title

TITLE A descriptive title, used on the printed output and in the CONTOUR plot file, up to 72 characters long. This is a character variable.

3.1.3 Namelist RSTRT

The parameters in this namelist control the use of the restart option. The contents of the restart files are described in Section 4.4.

IREST 0 if no restart files are to be read or written.
 1 to write restart files at the end of the calculation.
 2 to read restart files for initial values, and to write restart files at the end of the calculation. For turbulent flow, the current calculation must use the same turbulence model as the previous calculation. Note that only the initial values and the computational mesh are read from restart files. The usual namelist input must still be read in. Of course, some input parameters, such as the reference conditions or those specifying the grid, must not be changed during a restart.
 3 same as the IREST = 2 option, but for cases in which the previous calculation was either laminar or used an algebraic turbulence model, and the current calculation uses a two-equation turbulence model.

Two restart files are written and/or read. One contains the computational mesh, and the other contains the mean flow field and the k and ϵ values (if a two-equation turbulence model is being used.) For IREST = 0 or 1, the initial mean flow field will be generated in subroutine INIT, and the initial k and ϵ values (if necessary) will be generated in subroutine KEINIT. The default value is 0.

NRQIN Unit number for reading the restart flow field. The default value is 11.

NRQOUT Unit number for writing the restart flow field. The default value is 12.

NRXIN Unit number for reading the restart computational mesh. The default value is 13.

NRXOUT Unit number for writing the restart computational mesh. The default value is 14.

3.1.4 Namelist IO

Printout Controls

The following parameters specify which variables are to be printed, and at what locations in both time and space.

IVOUT An array of up to 50 elements specifying which variables are to be printed. The variables currently available for printing are listed and defined in Table 3-3.⁴ The default values are 1, 2, 20, 30, 40, 45*0, corresponding to printout of x and y -velocity, and static density, pressure, and temperature.

IWOUT1 A 2-element array, given as IWOUT1(IBOUND), indicating whether or not various parameters are to be printed along the ξ boundaries. The subscript IBOUND = 1 or 2, corresponding to the $\xi = 0$ and $\xi = 1$ boundaries, respectively. Valid values of IWOUT1(IBOUND) are:

0 for no printout.

⁴ The definitions of k_t and k_r in Table 3-3 (IVOUT = 92 and 102) assume a constant turbulent Prandtl number is being specified in namelist TURB.

- 1 for printout along the boundary, with normal derivatives computed using three-point one-sided differencing.
- 1 for printout along the boundary, with normal derivatives computed using two-point one-sided differencing.

The default values are both 0.

IWOUT2

A 2-element array, given as IWOUT2(IBOUND), indicating whether or not various parameters are to be printed along the η boundaries. The subscript IBOUND = 1 or 2, corresponding to the $\eta = 0$ and $\eta = 1$ boundaries, respectively. Valid values of IWOUT2(IBOUND) are:

- 0 for no printout.
- 1 for printout along the boundary, with normal derivatives computed using three-point one-sided differencing.
- 1 for printout along the boundary, with normal derivatives computed using two-point one-sided differencing.

The default values are both 0.

The parameters printed via the IWOUT1 and/or IWOUT2 input arrays are the Cartesian coordinates x and y , the static pressure p , the skin friction coefficient c_f , the shear stress τ_w , the static temperature T , the heat transfer coefficient h , the heat flux q_w , and the Stanton number St . See Section 4.1 for the definitions of these parameters. Note that some of these parameters are meaningful only if the boundary is a solid wall. In general, using three-point one-sided differences to compute normal derivatives will be more accurate. However, for nearly-separated flows, it has been found that three-point differencing can give misleading negative values of τ_w and c_f . The parameters are printed at every boundary point, not just at the points specified by the input parameters IPRT1 and IPRT2, or IPRT1A and IPRT2A.

IDEBUG

An array of up to 20 elements used to turn on additional printout, normally used for debugging purposes. Except where noted, set IDEBUG(I) = 1 for printout number I. For options 1 through 8, the input parameters IPRT1 and IPRT2, or IPRT1A and IPRT2A, determine the grid points at which the printout appears. Note that some of these options can generate a lot of output. Judicious use of the "IPRT" controls is recommended. The debug options currently available are as follows:

Number Printout

- 1 Coefficient block submatrices and source term subvectors at time level $n = \text{IDEBUG}(1)$ if $\text{IDEBUG}(1) > 0$, or at time levels $n \geq |\text{IDEBUG}(1)|$ if $\text{IDEBUG}(1) < 0$. This printout is done after the elimination of any off-diagonal boundary condition submatrices (subroutine BCELIM) and after any artificial viscosity has been added (subroutine AVISC1 or 2), but before any rearrangement of the elements in the boundary condition submatrices (subroutine FILTER.)
- 2 Coefficient block submatrices and source term subvectors at time level $n = \text{IDEBUG}(2)$ if $\text{IDEBUG}(2) > 0$, or at time levels $n \geq |\text{IDEBUG}(2)|$ if $\text{IDEBUG}(2) < 0$. This printout is done after the elimination of any off-diagonal boundary condition submatrices (subroutine BCELIM), but before any artificial viscosity has been added (subroutine AVISC1 or 2) and before any rearrangement of the elements in the boundary condition submatrices (subroutine FILTER.)

Number Printout

- 3 Boundary condition coefficient block submatrices and source term subvectors at time level $n = \text{IDEBUG}(3)$ if $\text{IDEBUG}(3) > 0$, or at time levels $n \geq |\text{IDEBUG}(3)|$ if $\text{IDEBUG}(3) < 0$. This printout is done before the elimination of any off-diagonal boundary condition submatrices (subroutine BCELIM) and before any rearrangement of the elements in the boundary condition submatrices (subroutine FILTER.)
- 4 Boundary condition coefficient block submatrices and source term subvectors at time level $n = \text{IDEBUG}(4)$ if $\text{IDEBUG}(4) > 0$, or at time levels $n \geq |\text{IDEBUG}(4)|$ if $\text{IDEBUG}(4) < 0$. This printout is done after the elimination of any off-diagonal boundary condition submatrices (subroutine BCELIM) and after any rearrangement of the elements in the boundary condition submatrices (subroutine FILTER.)
- 5 Intermediate solution Q^* after the first ADI sweep at time level $n = \text{IDEBUG}(5)$ if $\text{IDEBUG}(5) > 0$, or at time levels $n \geq |\text{IDEBUG}(5)|$ if $\text{IDEBUG}(5) < 0$.
- 6 Final solution Q^* after the last ADI sweep at time level $n = \text{IDEBUG}(6)$ if $\text{IDEBUG}(6) > 0$, or at time levels $n \geq |\text{IDEBUG}(6)|$ if $\text{IDEBUG}(6) < 0$.
- 7 Cartesian coordinates, metric coefficients, and inverse of the grid transformation Jacobian computed in subroutine METS.
- 8 Baldwin-Lomax turbulence model parameters at time level $n = \text{IDEBUG}(8)$ if $\text{IDEBUG}(8) > 0$, or at time levels $n \geq |\text{IDEBUG}(8)|$ if $\text{IDEBUG}(8) < 0$.

The default values are all 0.

IUNITS 0 for input and output in English units.
 1 for input and output in SI units.

The default value is 0.

IPRT Results are printed every IPRT'th time level. However, the initial and final flow fields are always printed. The default value is 1.

IPRTA An array of up to 101 elements specifying the time levels at which results are to be printed. The initial conditions are at time level 1. If the calculation converges, or if the pressure or temperature is non-positive, the results are printed regardless of the value of IPRTA. If this parameter is specified, it overrides the value of IPRT. The default values are all 0.

IPRT1 Flow field results are printed at every IPRT1'th grid point in the ξ direction. However, the results at the boundaries are always printed. The default value is 1.

IPRT2 Flow field results are printed at every IPRT2'th grid point in the η direction. However, the results at the boundaries are always printed. The default value is 1.

IPRT1A An array of up to N1 elements (see Namelist NUM) specifying the ξ indices at which flow field results are to be printed. The indices must be specified in ascending order. If this parameter is specified, it overrides the value of IPRT1. The default values are all 0.

IPRT2A An array of up to N2 elements (see Namelist NUM) specifying the η indices at which flow field results are to be printed. The indices must be specified in ascending order. If this parameter is specified, it overrides the value of IPRT2. The default values are all 0.

NHMAX Maximum number of time levels allowed in the printout of the convergence history file (not counting the first two, which are always printed.) The default value is 100.

Plot File Controls

In addition to the printed output, files called plot files may be written for use by various post-processing routines. The following parameters specify the type of plot files to be written, and at what locations in both time and space. These plot files are described in greater detail in Section 4.2.

IPLOT 0 for no plot file.
 1 to write results into an auxiliary file, in CONTOUR format, for later post-processing. If multiple time levels are to be written into the file, they will be stacked sequentially. The value of the time τ will not be written into the file.⁵
 - 1 to write results into an auxiliary file in CONTOUR format. For multiple time levels, $\tau_{i,j}$ will be stored in the z slot. (The subscripts i and j represent grid point indices in the ξ and η directions.)
 2 to write results into auxiliary files, in PLOT3D/WHOLE format. Multiple time levels will be stacked sequentially,⁶ with $\tau_{1,1}$ stored in the Q file header.⁷
 3 to write results into auxiliary files, in PLOT3D/PLANES format. Multiple time levels will be stacked sequentially,⁶ with $\tau_{1,1}$ stored in the Q file header.⁷ Since *Proteus 2-D* is two-dimensional, the IPLOT = 3 option creates XYZ and Q files identical to those created using the IPLOT = 2 option.
 - 3 to write results into auxiliary files, in PLOT3D/PLANES format. For multiple time levels, $\tau_{i,j}$ will be stored in the z slot in the XYZ file.
 4 to write results into auxiliary files, in PLOT2D format. Multiple time levels will be stacked sequentially,⁶ with $\tau_{1,1}$ stored in the Q file header.⁷

The default value is 0.

IPLT Results are written into the plot file every IPLTth time level. However, if IPLT > 0, the initial and final flow fields are automatically included in the file. If IPLT = 0, only the final flow field is written into the file. The default value is 0.

IPLTA An array of up to 101 elements specifying the time levels at which results are to be written into the plot file. The initial conditions are at time level 1. If the calculation converges, or if the pressure or temperature is non-positive, the results are written into the plot file regardless of the value of IPLTA. If this parameter is specified, it overrides the value of IPLT. The default values are all 0.

Unit Numbers

The following parameters specify the Fortran unit numbers used for various input and output files. NIN, the unit number for reading the standard input file, is hardwired in the program as 5.

NOUT Unit number for printing standard output. The default value is 6.

NGRID Unit number for reading computational coordinate system file. The default value is 7.

⁵ The IPLOT = - 1 option is the better one to use for CONTOUR plot files. The IPLOT = 1 option is included only to be consistent with the various PLOT3D and PLOT2D options.

⁶ The current version of PLOT3D does not work for multiple time levels, although future versions might. You can, however, fake it out using the IPLOT = - 3 option.

⁷ Note that with IDTAU = 5 or 6, τ will vary in space, and therefore $\tau_{i,j} \neq \tau_{1,1}$.

NPLOTX	Unit number for writing XYZ file when using PLOT3D or PLOT2D plot file format. The default value is 8.
NPLOT	Unit number for writing CONTOUR plot file, or for writing Q file when using PLOT3D or PLOT2D format. The default value is 9.
NHIST	Unit number for writing convergence history file. The default value is 10.
NSCR1	Unit number for scratch file used in subroutine PLOT when IPLOT = - 3. The default value is 20.

3.1.5 Namelist GMTRY

Coordinate System Type

These parameters specify the type of flow domain being analyzed. Simple Cartesian or polar configurations can be done automatically. For more complex geometries, the configuration is determined by reading a pre-stored coordinate file. Note however, that the number of grid points and their distribution can be changed by the parameters in namelist NUM.

IAXI	0 for a two-dimensional planar calculation. 1 for an axisymmetric calculation.
	The default value is 0.
NGEOM	Flag used to specify type of computational coordinates. Currently coded are:
	1 Cartesian (x-y) computational coordinates.
	2 Polar (r' - θ') computational coordinates. ⁸
	10 Get computational coordinates from coordinate system file. The contents of this file are described in Section 3.2.

The default value is 1.

Cartesian Computational Coordinates

The following parameters specify the size of the flow domain for the Cartesian coordinate option (NGEOM = 1). The computational (ξ, η) domain for this option is shown in physical (x,y) space in Figure 3.1.

XMIN	Minimum x-coordinate for Cartesian coordinate option. The default value is 0.0.
XMAX	Maximum x-coordinate for Cartesian coordinate option. The default value is 1.0.
YMIN	Minimum y-coordinate for Cartesian coordinate option. The default value is 0.0.
YMAX	Maximum y-coordinate for Cartesian coordinate option. The default value is 1.0.

⁸ There may be some confusion between the axisymmetric flow option and the polar coordinate system option, or between the axisymmetric radius r and the polar coordinate r' . They are not the same thing. The governing flow equations were developed by originally writing them in Cartesian (x-y) coordinates, then transforming them into generalized (ξ, η) coordinates. Therefore, any computational coordinate system that is used, including the polar coordinate system, must be related to the original Cartesian system through the transformation metrics and Jacobian. The parameters r' and θ' are used only to initially define the coordinates in the NGEOM = 2 option. Now, if the (x-y) coordinates, no matter how they are obtained, are rotated about the Cartesian x axis, the result is a cylindrical coordinate system with y representing the radius r . Thus, the axisymmetric flow option can be used with any of the coordinate system options. The polar coordinate option would be useful, for instance, for flow over a sphere.

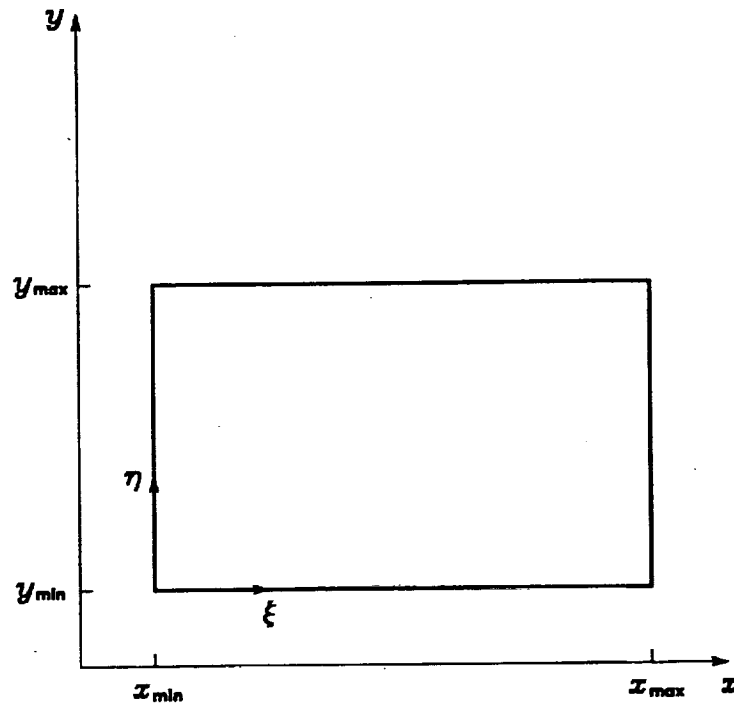


Figure 3.1 - Cartesian computational coordinates.

Polar Computational Coordinates

The following parameters specify the size of the flow domain for the polar coordinate option (NGEOM = 2). The computational (ξ, η) domain for this option is shown in physical (x, y) space in Figure 3.2.

RMIN	Minimum r' -coordinate for polar coordinate option. The default value is 0.0.
RMAX	Maximum r' -coordinate for polar coordinate option. The default value is 1.0.
THMIN	Minimum θ' -coordinate in degrees for polar coordinate option. The default value is 0.0.
THMAX	Maximum θ' -coordinate in degrees for polar coordinate option. The default value is 90.0.

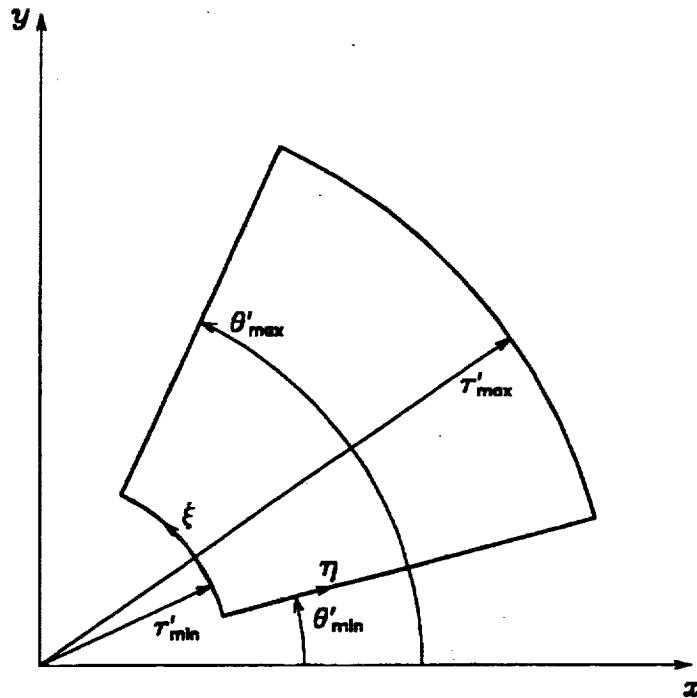


Figure 3.2 - Polar computational coordinates.

3.1.6 Namelist FLOW

Control Flags

The following parameters are flags that specify the type of equations to be solved, and which variables are being supplied as initial conditions.

IEULER 0 for a full time-averaged Navier-Stokes calculation.
 1 for an Euler calculation (i.e., neglecting all viscous and heat conduction terms.)

The default value is 0.

ITHIN A 2-element array, specified as ITHIN(IDIR), indicating whether or not the thin-layer option is to be used in direction IDIR. The subscript IDIR = 1 or 2, corresponding to the ξ and η directions, respectively. Valid values of ITHIN(IDIR) are:

0 to include second derivative viscous terms in direction IDIR.
 1 to use the thin-layer option in direction IDIR. This does not decrease the execution time much, but may be useful if the grid in direction IDIR is not sufficiently dense to resolve second derivatives in that direction.

The default values are both 0.

IHSTAG 0 to solve the energy equation. The dimensioning parameter NEQP must be at least 4 (if ISWIRL = 0) or 5 (if ISWIRL = 1). (See Section 6.2.)
 1 to eliminate the energy equation by assuming constant stagnation enthalpy per unit mass. This significantly lowers the overall execution time.

The default value is 0.

- ILAMV 0 for constant laminar viscosity and thermal conductivity coefficients equal to MUR and KTR.
 1 for variable laminar viscosity and thermal conductivity coefficients, computed as a function of local temperature using Sutherland's formula for air (White, 1974).

The default value is 0.

- ISWIRL 0 for no swirl.
 1 for a swirling calculation in axisymmetric flow. The dimensioning parameter NEQP must be at least 4 (if IHSTAG = 1) or 5 (if IHSTAG = 0). (See Section 6.2.)

The default value is 0.

ICVARS Parameter specifying which variables are being supplied as initial conditions for the time marching procedure by subroutine INIT. Remember that the initial conditions must be nondimensionalized by the reference conditions listed in Table 3-2. (See Section 5.0 for details on defining initial conditions.) When the energy equation is being solved (IHSTAG = 0), and the flow is two-dimensional, or axisymmetric without swirl (IAXI = 0 or ISWIRL = 0), the allowed values are:

ICVARS Variables Supplied By INIT

1	$\rho, \rho u, \rho v, E_T$
2	p, u, v, T
3	ρ, u, v, T
4	p, u, v, ρ
5	c_p, u, v, T
6	p, M, α_v, T

When the energy equation is being solved (IHSTAG = 0), and the flow is axisymmetric with swirl (IAXI = 1 and ISWIRL = 1), the allowed values are:

ICVARS Variables Supplied By INIT

1	$\rho, \rho u, \rho v, \rho w, E_T$
2	p, u, v, w, T
3	ρ, u, v, w, T
4	p, u, v, w, ρ
5	c_p, u, v, w, T
6	$p, M, \alpha_v, \alpha_w, T$

When constant stagnation enthalpy is assumed (IHSTAG = 1), and the flow is two-dimensional, or axisymmetric without swirl (IAXI = 0 or ISWIRL = 0), the allowed values are:

ICVARS Variables Supplied By INIT

1	$\rho, \rho u, \rho v$
2	p, u, v
3	ρ, u, v
5	c_p, u, v
6	p, M, α_v

When constant stagnation enthalpy is assumed (IHSTAG = 1), and the flow is axisymmetric with swirl (IAXI = 1 and ISWIRL = 1), the allowed values are:

ICVARS Variables Supplied By INIT

1	$\rho, \rho u, \rho v, \rho w$
2	p, u, v, w
3	ρ, u, v, w
5	c_p, u, v, w
6	p, M, α_v, α_w

In the above tables, c_p , α_v , and α_w represent static pressure coefficient, flow angle in degrees in the x - y (or x - r) plane, and flow angle in degrees in the x - θ plane, respectively. The default value is 2.

Reference Conditions

The following parameters specify the six basic reference conditions for length, velocity, temperature, density, viscosity, and thermal conductivity. These reference conditions are used, along with some additional reference conditions derived from them, as the nondimensionalizing factors for nondimensional input and output parameters. The dimensional reference conditions may be read in using either English or SI units, depending on the value of IUNITS.

LR	Reference length L_r in feet (meters). This is a real variable. The default value is 1.0.
UR	Reference velocity u_r in ft/sec (m/sec). Either UR or MACHR may be specified, but not both. The unspecified one will be computed from the remaining reference conditions. The default value is $a_r = (\gamma_r \bar{R} T_r)^{1/2}$, the speed of sound at the reference temperature.
MACHR	Reference Mach number, $M_r = u_r / (\gamma_r \bar{R} T_r)^{1/2}$. Either MACHR or UR may be specified, but not both. The unspecified one will be computed from the remaining reference conditions. This is a real variable. The default value is 0.0.
TR	Reference temperature T_r in °R (K). The default value is 519.0 °R (288.333 K).
RHOR	Reference density ρ_r in lb _m /ft ³ (kg/m ³). The default value is 0.07645 lb _m /ft ³ (1.22461 kg/m ³).
MUR	Reference viscosity μ_r in lb _m /ft-sec (kg/m-sec). Either MUR or RER may be specified, but not both. The unspecified one will be computed from the remaining reference conditions. This is a real variable. The default value is the viscosity for air at the reference temperature TR.
RER	Reference Reynolds number, $Re_r = \rho_r u_r L_r / \mu_r$. Either RER or MUR may be specified, but not both. The unspecified one will be computed from the remaining reference conditions. The default value is 0.0.
KTR	Reference thermal conductivity k_r in lb _m -ft/sec ³ -°R (kg-m/sec ³ -K). Either KTR or PRLR may be specified, but not both. The unspecified one will be computed from the remaining reference conditions. This is a real variable. The default value is the thermal conductivity for air at the reference temperature TR.
PRLR	Reference laminar Prandtl number $Pr_{lr} = c_{p,r} \mu_r / k_r$. Here $c_{p,r}$ is the specific heat at constant pressure, defined as either $c_{p,r}(T_r)$ or $\gamma_r \bar{R} / (\gamma_r - 1)$, depending on whether or not a value is being specified for the input parameter GAMR. Either PRLR or KTR may be specified, but not both. The unspecified one will be computed from the remaining reference conditions. The default value is 0.0.

Fluid Properties

The following parameters provide information about the fluid being used.

RG	Gas constant \bar{R} in $\text{ft}^2/\text{sec}^2\text{-}^\circ\text{R}$ ($\text{m}^2/\text{sec}^2\text{-K}$). The default value is $1716 \text{ ft}^2/\text{sec}^2\text{-}^\circ\text{R}$ ($286.96 \text{ m}^2/\text{sec}^2\text{-K}$).
GAMR	Reference ratio of specific heats, $\gamma_r = c_{p,r}/c_{v,r}$. This parameter acts as a flag for a constant specific heat option. If a non-zero value for GAMR is specified by the user, the specific heats c_p and c_v are computed from GAMR and RG, and treated as constants. Otherwise they are computed locally as a function of temperature. The default value is 0.0.
HSTAGR	Stagnation enthalpy h_T in ft^2/sec^2 (m^2/sec^2). This parameter is only used with the constant stagnation enthalpy option (IHSTAG = 1). The default value is computed from the reference conditions.

3.1.7 Namelist BC

The parameters in this namelist specify the boundary conditions to be used for the mean flow equations and, if necessary, for the $k\text{-}\epsilon$ turbulence model equations. For the mean flow, NEQ conditions must be specified at each computational boundary, where NEQ is the number of coupled equations being solved. NEQ will be equal to 3, 4, or 5 depending on the values of IHSTAG and ISWIRL. (See Table 3-4.)

Note that the boundary conditions may be thought of as simply NEQ additional equations to be solved on the boundary. They do not necessarily have to be associated one-to-one with the governing differential equations or the dependent variables. They must, however, be functions of the dependent variables and sufficiently complete to set constraints on each of the dependent variables through their functional form. They must also, of course, be independent of one another and physically appropriate for the problem being solved.

Three different methods are available for setting boundary conditions for steady flow computations. The first, and easiest, way is to specify the type of boundary (i.e., solid wall, symmetry, etc.) using the KBC input parameters. These parameters act as "meta" flags, triggering the automatic setting of the necessary NEQ individual boundary conditions at the specified boundary.

Second, if more flexibility is needed, the NEQ individual boundary conditions may be set for each boundary using the JBC and GBC input parameters. The boundary condition *type* (specified value, specified gradient, etc.) is given by JBC, and the boundary condition *value* by GBC. With these parameters, the same conditions are applied over the entire surface.

And third, if even greater flexibility is needed, the NEQ individual boundary conditions may be set for each boundary using the IBC and FBC input parameters. These are analogous to the JBC and GBC parameters (i.e., the boundary condition type is given by IBC, and the value by FBC), but they allow a point-by-point distribution of type and value to be specified instead of using the same type and value over the entire surface.⁹

For a given boundary, boundary conditions specified via the KBC parameters override those specified using the JBC and GBC parameters, which in turn override those specified using the IBC and FBC parameters. However, the different methods may be used in combination as long as they don't conflict. For example, the KBC parameters may be used for two boundaries, the JBC and GBC parameters for the third boundary, and the IBC and FBC parameters for the fourth boundary. And, on a single boundary, the JBC and GBC parameters may be used for some of the NEQ boundary conditions, and the IBC and FBC parameters for the rest.

⁹ However, note that a specified point-by-point distribution of a function value is most easily set using the "no change from initial conditions" option with the JBC parameters.

Unsteady boundary conditions may be used when individual boundary conditions are specified for the entire surface, but not when boundary conditions are specified point-by-point.

With one exception, the NEQ boundary conditions at each boundary may be specified in any order. The exception is any condition on one of the dependent conservation variables Q . These must be specified in the order given in Table 3-4.

If a problem requires a boundary condition of the form $\Delta F = 0$, $F = f$, $\partial F / \partial \phi = f$, or $\nabla F \cdot \bar{n} = f$, where F is not one of the functions already built into *Proteus*, the subroutines BCF and BCFLIN may be used. This requires that the user supply subroutine BCFLIN. A test case with a user-written version of BCFLIN is presented in Section 9.2. Subroutines BCF and BCFLIN are described in detail in Volume 3.

If the Chien k - ϵ turbulence model is being used, boundary conditions for k and ϵ must be specified in addition to those specified for the mean flow equations. (Unless a spatially periodic boundary condition is being used for the mean flow. In this case, separate boundary conditions for k and ϵ are not needed.) Surface boundary conditions may be specified using the JBCT/GBCT parameters, or point-by-point boundary conditions may be specified using the IBCT/FBCT parameters. These parameters are analogous to the JBC/GBC and IBC/FBC parameters used for the mean flow boundary conditions. There are no input parameters for k - ϵ boundary conditions that are analogous to the KBC parameters.

Mean Flow Boundary Types with KBC

The following parameters set mean flow boundary conditions by specifying the type of boundary (i.e., solid wall, symmetry, etc.). These parameters act as "meta" flags, triggering the automatic setting of the necessary JBC and GBC values.

- KBC1 An array, given as KBC1(IBOUND), specifying the types of boundaries in the ξ direction. The subscript IBOUND = 1 or 2, corresponding to the $\xi = 0$ and $\xi = 1$ boundaries, respectively. The default values are both 0.
- KBC2 An array, given as KBC2(IBOUND), specifying the types of boundaries in the η direction. The subscript IBOUND = 1 or 2, corresponding to the $\eta = 0$ and $\eta = 1$ boundaries, respectively. The default values are both 0.

The boundary types that may be specified are described briefly in the following table, and in greater detail in Table 3-5. For boundary types involving gradient boundary conditions, 2-point differencing is used if the input KBC value is positive, and 3-point differencing is used if it is negative. For boundary types involving "no change from initial conditions"-type boundary conditions (e.g., $\Delta T = 0$), the proper boundary values must be set in the initial conditions. When the KBC parameters are used, the corresponding GBC and FBC parameters should be defaulted.

<u>KBC Value</u>	<u>Boundary Type</u>
± 1	No-slip adiabatic wall.
± 2	No-slip wall, specified temperature.
± 3	Inviscid wall.
10	Subsonic inflow, linear extrapolation.
± 11	Subsonic inflow, zero gradient.
20	Subsonic outflow, linear extrapolation.
± 21	Subsonic outflow, zero gradient.
30	Supersonic inflow.
40	Supersonic outflow, linear extrapolation.
± 41	Supersonic outflow, zero gradient.
± 50	Symmetry.
60	Spatially periodic.

Boundary conditions specified using the KBC parameter for a given boundary override any boundary condition types specified for that boundary using the JBC or IBC parameters. Note, however, that since

the default values for the KBC parameters are all 0, the default procedure for specifying boundary conditions is by using the JBC and GBC parameters.

Surface Mean Flow Boundary Condition Types and Values with JBC and GBC

The following parameters set the NEQ individual mean flow boundary condition types and values for each boundary using the JBC and GBC parameters. With these parameters, the same conditions are applied over the entire boundary. Remember that the boundary condition values must be nondimensionalized by the reference conditions listed in Table 3-2. If some of the boundary conditions are being specified using the IBC and FBC parameters, the appropriate JBC parameters must be set equal to -1 , as described below.

- | | |
|------|---|
| JBC1 | A two-dimensional array, given as JBC1(IEQ,IBOUND), specifying the type of boundary conditions to be used on the $\xi = 0$ and $\xi = 1$ boundaries. Here IEQ = 1 to NEQ corresponding to each equation, and IBOUND = 1 or 2 corresponding to the $\xi = 0$ and $\xi = 1$ boundaries, respectively. Setting JBC1 = -1 signals the code to use boundary conditions specified point-by-point, as given by the input arrays IBC1 and FBC1. See Table 3-6 for a list of allowed boundary condition types. The default values are all 0. |
| JBC2 | A two-dimensional array, given as JBC2(IEQ,IBOUND), specifying the type of boundary conditions to be used on the $\eta = 0$ and $\eta = 1$ boundaries. Here IEQ = 1 to NEQ corresponding to each equation, and IBOUND = 1 or 2 corresponding to the $\eta = 0$ and $\eta = 1$ boundaries, respectively. Setting JBC2 = -1 signals the code to use boundary conditions specified point-by-point, as given by the input arrays IBC2 and FBC2. See Table 3-6 for a list of allowed boundary condition types. The default values are all 0. |
| GBC1 | A two-dimensional array, given as GBC1(IEQ,IBOUND), specifying the values for the steady boundary conditions to be used on the $\xi = 0$ and $\xi = 1$ boundaries. Here IEQ = 1 to NEQ corresponding to each equation, and IBOUND = 1 or 2 corresponding to the $\xi = 0$ and $\xi = 1$ boundaries, respectively. The default values are all 0.0. |
| GBC2 | A two-dimensional array, given as GBC2(IEQ,IBOUND), specifying the values for the steady boundary conditions to be used on the $\eta = 0$ and $\eta = 1$ boundaries. Here IEQ = 1 to NEQ corresponding to each equation, and IBOUND = 1 or 2 corresponding to the $\eta = 0$ and $\eta = 1$ boundaries, respectively. The default values are all 0.0. |

Note that boundary condition types 2, 12, 22, etc., are specified values of the derivative with respect to the computational coordinate, not with respect to the physical distance in the direction of the computational coordinate. See Section 6.3 of Volume 1 for details.

Note also that normal derivative boundary condition values are positive in the direction of increasing ξ or η . Thus, a positive value for GBC at $\xi = 0$ or $\eta = 0$ implies a flux into the computational domain, and a positive GBC at $\xi = 1$ or $\eta = 1$ implies a flux out of the computational domain. See Section 6.4 of Volume 1 for details. Similarly, the normal velocity V_n is positive in the direction of increasing ξ or η . Thus, a positive V_n at $\xi = 0$ or $\eta = 0$ implies flow into the computational domain, and a positive V_n at $\xi = 1$ or $\eta = 1$ implies flow out of the computational domain. See the description of subroutine BCVDIR in Section 4.2 of Volume 3 for details.

Boundary conditions specified using the JBC and GBC parameters for given values of IEQ and IBOUND override any boundary conditions specified for those values of IEQ and IBOUND using the IBC and FBC parameters. Note that since the default values for the JBC parameters are all 0, the default boundary conditions are "no change from initial conditions" for the conservation variables.

Point-by-Point Mean Flow Boundary Condition Types and Values with IBC and FBC

The following parameters set the NEQ individual mean flow boundary condition types and values for each boundary using the IBC and FBC parameters. With these parameters, point-by-point distributions are specified on the boundary for the boundary condition types and values. Remember that the boundary condition values must be nondimensionalized by the reference conditions listed in Table 3-2. Note that these parameters are activated by setting the appropriate JBC parameters equal to -1, as described below.

- IBC1 A three-dimensional array, given as IBC1(I2,IEQ,IBOUND), specifying the type of boundary conditions to be used on the $\xi = 0$ and $\xi = 1$ boundaries. Here I2 = 1 to N2 corresponding to each grid point on the boundary, IEQ = 1 to NEQ corresponding to each equation, and IBOUND = 1 or 2 corresponding to the $\xi = 0$ and $\xi = 1$ boundaries, respectively. JBC1(IEQ,IBOUND) must be set equal to -1. See Table 3-6 for a list of allowed boundary condition types. The default values are all 0.
- IBC2 A three-dimensional array, given as IBC2(I1,IEQ,IBOUND), specifying the type of boundary conditions to be used on the $\eta = 0$ and $\eta = 1$ boundaries. Here I1 = 1 to N1 corresponding to each grid point on the boundary, IEQ = 1 to NEQ corresponding to each equation, and IBOUND = 1 or 2 corresponding to the $\eta = 0$ and $\eta = 1$ boundaries, respectively. JBC2(IEQ,IBOUND) must be set equal to -1. See Table 3-6 for a list of allowed boundary condition types. The default values are all 0.
- FBC1 A three-dimensional array, given as FBC1(I2,IEQ,IBOUND), specifying the values for the steady boundary conditions to be used on the $\xi = 0$ and $\xi = 1$ boundaries. Here I2 = 1 to N2 corresponding to each grid point on the boundary, IEQ = 1 to NEQ corresponding to each equation, and IBOUND = 1 or 2 corresponding to the $\xi = 0$ and $\xi = 1$ boundaries, respectively. The default values are all 0.0.
- FBC2 A three-dimensional array, given as FBC2(I1,IEQ,IBOUND), specifying the values for the steady boundary conditions to be used on the $\eta = 0$ and $\eta = 1$ boundaries. Here I1 = 1 to N1 corresponding to each grid point on the boundary, IEQ = 1 to NEQ corresponding to each equation, and IBOUND = 1 or 2 corresponding to the $\eta = 0$ and $\eta = 1$ boundaries, respectively. The default values are all 0.0.

Note that boundary condition types 2, 12, 22, etc., are specified values of the derivative with respect to the computational coordinate, not with respect to the physical distance in the direction of the computational coordinate. See Section 6.3 of Volume 1 for details.

Note also that normal derivative boundary condition values are positive in the direction of increasing ξ or η . Thus, a positive value for FBC at $\xi = 0$ or $\eta = 0$ implies a flux into the computational domain, and a positive FBC at $\xi = 1$ or $\eta = 1$ implies a flux out of the computational domain. See Section 6.4 of Volume 1 for details. Similarly, the normal velocity V_n is positive in the direction of increasing ξ or η . Thus, a positive V_n at $\xi = 0$ or $\eta = 0$ implies flow into the computational domain, and a positive V_n at $\xi = 1$ or $\eta = 1$ implies flow out of the computational domain. See the description of subroutine BCVDIR in Section 4.2 of Volume 3 for details.

Unsteady Mean Flow Boundary Conditions

The following parameters are used to specify unsteady mean flow boundary conditions. The boundary condition type (specified value, specified gradient, etc.) is given by JBC, as described above, but the value is given by GTBC. The type of unsteadiness (general or periodic) is given by JTBC.

- JTBC1 A two-dimensional array, given as JTBC1(IEQ,IBOUND), specifying the type of time dependency for the boundary conditions on the $\xi = 0$ and $\xi = 1$ boundaries. Here IEQ = 1 to NEQ corresponding to each equation, and IBOUND = 1 or 2

corresponding to the $\xi = 0$ and $\xi = 1$ boundaries, respectively. Valid values of JTBC1(IEQ,IBOUND) are:

- 0 for a steady boundary condition, whose value is given by GBC1.
- 1 for a general unsteady boundary condition, whose value is determined by linear interpolation in the input table of GTBC1 vs. NTBCA.
- 2 for a time-periodic boundary condition of the form $g_1 + g_2 \sin(g_3 n + g_4)$, where n is the time level and g_1 through g_4 are given by the first four values of GTBC1.

The default values are all 0.

JTBC2 A two-dimensional array, given as JTBC2(IEQ,IBOUND), specifying the type of time dependency for the boundary conditions on the $\eta = 0$ and $\eta = 1$ boundaries. Here IEQ = 1 to NEQ corresponding to each equation, and IBOUND = 1 or 2 corresponding to the $\eta = 0$ and $\eta = 1$ boundaries, respectively. Valid values of JTBC2(IEQ,IBOUND) are:

- 0 for a steady boundary condition, whose value is given by GBC2.
- 1 for a general unsteady boundary condition, whose value is determined by linear interpolation in the input table of GTBC2 vs. NTBCA.
- 2 for a time-periodic boundary condition of the form $g_1 + g_2 \sin(g_3 n + g_4)$, where n is the time level and g_1 through g_4 are given by the first four values of GTBC2.

The default values are all 0.

NTBC Number of values in the tables of GTBC1 and/or GTBC2 vs. NTBCA for the general unsteady boundary condition option. The maximum value allowed is the value of the dimensioning parameter NTP. (See Section 6.2.) The default value is 0.

NTBCA An array of NTBC time levels at which GTBC1 and/or GTBC2 are specified for the general unsteady boundary condition option. The default values are all 0.

GTBC1 A three-dimensional array, given as GTBC1(ITBC,IEQ,IBOUND), used in the unsteady and time-periodic boundary condition options for the $\xi = 0$ and $\xi = 1$ boundaries. Here IEQ = 1 to NEQ corresponding to each equation, and IBOUND = 1 or 2 corresponding to the $\xi = 0$ and $\xi = 1$ boundaries, respectively. For general unsteady boundary conditions the subscript ITBC = 1 to NTBC, corresponding to the time levels in the array NTBCA, and GTBC1 specifies the boundary condition value directly. For time-periodic boundary conditions the subscript ITBC = 1 to 4, and GTBC1 specifies the four coefficients in the equation used to determine the boundary condition value. The default values are all 0.0.

GTBC2 A three-dimensional array, given as GTBC2(ITBC,IEQ,IBOUND), used in the unsteady and time-periodic boundary condition options for the $\eta = 0$ and $\eta = 1$ boundaries. Here IEQ = 1 to NEQ corresponding to each equation, and IBOUND = 1 or 2 corresponding to the $\eta = 0$ and $\eta = 1$ boundaries, respectively. For general unsteady boundary conditions the subscript ITBC = 1 to NTBC, corresponding to the time levels in the array NTBCA, and GTBC2 specifies the boundary condition value directly. For time-periodic boundary conditions the subscript ITBC = 1 to 4, and GTBC2 specifies the four coefficients in the equation used to determine the boundary condition value. The default values are all 0.0.

k-ε Surface Boundary Condition Types and Values with JBCT and GBCT

The following parameters set the individual k and ϵ boundary condition types and values for each boundary using the JBCT and GBCT parameters. With these parameters, the same conditions are applied over the entire boundary. Remember that the boundary condition values must be nondimensionalized by the reference conditions listed in Table 3-2. None of the following parameters are needed if spatially peri-

odic boundary conditions are being used. If either of the boundary conditions for a computational boundary is being specified using the IBCT and FBCT parameters, the appropriate JBCT parameters must be set equal to -1 , as described below.

- JBCT1** A two-dimensional array, given as JBCT1(IEQ,IBOUND), specifying the type of boundary conditions to be used on the $\xi = 0$ and $\xi = 1$ boundaries. Here IEQ = 1 or 2 corresponding to k and ε , respectively, and IBOUND = 1 or 2 corresponding to the $\xi = 0$ and $\xi = 1$ boundaries, respectively. Setting JBCT1 = -1 signals the code to use boundary conditions specified point-by-point, as given by the input arrays IBCT1 and FBCT1. See Table 3-7 for a list of allowed boundary condition types. The default values are 0 for IEQ = 1, and 10 for IEQ = 2.
- JBCT2** A two-dimensional array, given as JBCT2(IEQ,IBOUND), specifying the type of boundary conditions to be used on the $\eta = 0$ and $\eta = 1$ boundaries. Here IEQ = 1 or 2 corresponding to k and ε , respectively, and IBOUND = 1 or 2 corresponding to the $\eta = 0$ and $\eta = 1$ boundaries, respectively. Setting JBCT2 = -1 signals the code to use boundary conditions specified point-by-point, as given by the input arrays IBCT2 and FBCT2. See Table 3-7 for a list of allowed boundary condition types. The default values are 0 for IEQ = 1, and 10 for IEQ = 2.
- GBCT1** A two-dimensional array, given as GBCT1(IEQ,IBOUND), specifying the values for the boundary conditions to be used on the $\xi = 0$ and $\xi = 1$ boundaries. Here IEQ = 1 or 2 corresponding to k and ε , respectively, and IBOUND = 1 or 2 corresponding to the $\xi = 0$ and $\xi = 1$ boundaries, respectively. The default values are all 0.0.
- GBCT2** A two-dimensional array, given as GBCT2(IEQ,IBOUND), specifying the values for the boundary conditions to be used on the $\eta = 0$ and $\eta = 1$ boundaries. Here IEQ = 1 or 2 corresponding to k and ε , respectively, and IBOUND = 1 or 2 corresponding to the $\eta = 0$ and $\eta = 1$ boundaries, respectively. The default values are all 0.0.

Note that boundary condition types 2 and 12 are specified values of the derivative with respect to the computational coordinate, not with respect to the physical distance in the direction of the computational coordinate. See Section 6.3 of Volume 1 for details.

Boundary conditions specified using the JBCT and GBCT parameters for given values of IEQ and IBOUND will override any boundary conditions specified for those values of IEQ and IBOUND using the IBCT and FBCT parameters. Note that the default values for the JBCT parameters are "no change from initial conditions" for the k and ε .

k- ε Point-by-Point Boundary Condition Types and Values with IBCT and FBCT

The following parameters set the individual k and ε boundary condition types and values for each boundary using the IBCT and FBCT parameters. With these parameters, point-by-point distributions are specified on the boundary for the boundary condition types and values. Remember that the boundary condition values must be nondimensionalized by the reference conditions listed in Table 3-2. None of the following parameters are needed if spatially periodic boundary conditions are being used. Note that these parameters are activated by setting the appropriate JBCT parameters equal to -1 , as described below.

- IBCT1** A three-dimensional array, given as IBCT1(I2,IEQ,IBOUND), specifying the type of boundary conditions to be used on the $\xi = 0$ and $\xi = 1$ boundaries. Here I2 = 1 to N2 corresponding to each grid point on the boundary, IEQ = 1 or 2 corresponding to k and ε , respectively, and IBOUND = 1 or 2 corresponding to the $\xi = 0$ and $\xi = 1$ boundaries, respectively. JBCT1(IEQ,IBOUND) must be set equal to -1 . See Table 3-7 for a list of allowed boundary condition types. The default values are 0 for IEQ = 1, and 10 for IEQ = 2.

- IBCT2** A three-dimensional array, given as IBCT2(I1,IEQ,IBOUND), specifying the type of boundary conditions to be used on the $\eta = 0$ and $\eta = 1$ boundaries. Here I1 = 1 to N1 corresponding to each grid point on the boundary, IEQ = 1 or 2 corresponding to k and ε , respectively, and IBOUND = 1 or 2 corresponding to the $\eta = 0$ and $\eta = 1$ boundaries, respectively. JBCT2(IEQ,IBOUND) must be set equal to -1. See Table 3-7 for a list of allowed boundary condition types. The default values are 0 for IEQ = 1, and 10 for IEQ = 2.
- FBCT1** A three-dimensional array, given as FBCT1(I2,IEQ,IBOUND), specifying the values for the boundary conditions to be used on the $\xi = 0$ and $\xi = 1$ boundaries. Here I2 = 1 to N2 corresponding to each grid point on the boundary, IEQ = 1 or 2 corresponding to k and ε , respectively, and IBOUND = 1 or 2 corresponding to the $\xi = 0$ and $\xi = 1$ boundaries, respectively. The default values are all 0.0.
- FBCT2** A three-dimensional array, given as FBCT2(I1,IEQ,IBOUND), specifying the values for the boundary conditions to be used on the $\eta = 0$ and $\eta = 1$ boundaries. Here I1 = 1 to N1 corresponding to each grid point on the boundary, IEQ = 1 or 2 corresponding to k and ε , respectively, and IBOUND = 1 or 2 corresponding to the $\eta = 0$ and $\eta = 1$ boundaries, respectively. The default values are all 0.0.

Note that boundary condition types 2 and 12 are specified values of the derivative with respect to the computational coordinate, not with respect to the physical distance in the direction of the computational coordinate. See Section 6.3 of Volume 1 for details.

3.1.8 Namelist NUM

Mesh Parameters

The following parameters specify the number of mesh points and the degree of packing.

- N1** Number of grid points N_1 in the ξ direction. For non-periodic boundary conditions in the ξ direction, the maximum value allowed is the value of the dimensioning parameter N1P. For spatially periodic boundary conditions, the maximum is N1P - 1. (See Section 6.2.) The default value is 5.
- N2** Number of grid points N_2 in the η direction. For non-periodic boundary conditions in the η direction, the maximum value allowed is the value of the dimensioning parameter N2P. For spatially periodic boundary conditions, the maximum is N2P - 1. (See Section 6.2.) The default value is 5.
- IPACK** A 2-element array, specified as IPACK(IDIR), indicating whether or not grid points are to be packed in direction IDIR. The subscript IDIR = 1 or 2, corresponding to the ξ and η directions, respectively. Valid values of IPACK(IDIR) are:
- 0 for no packing in direction IDIR.
 - 1 to pack points in direction IDIR using a transformation due to Roberts (1971). The location and amount of packing are specified by the array SQ.
- The default values are both 0.
- SQ** A two-dimensional array controlling the packing of grid points near computational boundaries, specified as SQ(IDIR,IPC). The subscript IDIR = 1 or 2 corresponding to packing in the ξ and η directions, respectively. The subscript IPC = 1 or 2, where SQ(IDIR,1) specifies the packing location, and SQ(IDIR,2) specifies the amount of packing.

Meaningful values for SQ(IDIR,1) are 0.0, 0.5, and 1.0, where 0.0 corresponds to packing near the lower boundary only (i.e., at ξ or $\eta = 0$, depending on IDIR), 1.0

corresponds to packing near the upper boundary only, and 0.5 corresponds to equal packing at both boundaries.

Meaningful values for SQ(IDIR,2) are values above 1.0, but generally 1.1 or below. The closer SQ(IDIR,2) is to 1.0, the tighter the packing will be.

The default values are SQ(IDIR,1) = 0.0 and SQ(IDIR,2) = 1.01 for IDIR = 1 and 2.

Artificial Viscosity Parameters

The following parameters specify the type and amount of artificial viscosity to be used.

- IAV4E 0 for no fourth-order explicit artificial viscosity.
 1 to include fourth-order explicit artificial viscosity using the constant coefficient model of Steger (1978).
 2 to include fourth-order explicit artificial viscosity using the nonlinear coefficient model of Jameson, Schmidt, and Turkel (1981).

The default value is 1.

- IAV2E 0 for no second-order explicit artificial viscosity.
 1 to include second-order explicit artificial viscosity using the constant coefficient model.
 2 to include second-order explicit artificial viscosity using the nonlinear coefficient model of Jameson, Schmidt, and Turkel (1981).

The default value is 0.

- IAV2I 0 for no second-order implicit artificial viscosity.
 1 to include second-order implicit artificial viscosity using the constant coefficient model of Steger (1978).

The default value is 1.

- CAVS4E For the constant coefficient model, CAVS4E(IEQ) specifies the fourth-order artificial viscosity coefficient $\varepsilon_4^{(p)}$ directly. For the nonlinear coefficient model it specifies the constant κ_4 . The subscript IEQ varies from 1 to NEQ corresponding to each coupled equation. (See Table 3-4 for the order of the equations being solved.) Good values for a given application are usually determined by experience, but recommended starting values are 1.0 for the constant coefficient model, 0.005 for the nonlinear model when spatially varying second-order time differencing is used, and 0.0002 for the nonlinear model when a spatially constant first-order time differencing is used. The default values are all 1.0.

- CAVS2E For the constant coefficient model, CAVS2E(IEQ) specifies the second-order artificial viscosity coefficient $\varepsilon_2^{(p)}$ directly. For the nonlinear coefficient model it specifies the constant κ_2 . The subscript IEQ varies from 1 to NEQ corresponding to each coupled equation. (See Table 3-4 for the order of the equations being solved.) Good values for a given application are usually determined by experience, but recommended starting values are 1.0 for the constant coefficient model, 0.01 for the nonlinear model for flows without shocks, and 0.1 for the nonlinear model for flows with shocks. The default values are all 1.0.

- CAVS2I Second-order implicit artificial viscosity coefficient, ε_i , specified as CAVS2I(IEQ). The subscript IEQ varies from 1 to NEQ corresponding to each coupled equation. (See Table 3-4 for the order of the equations being solved.) Good values for a given application are usually determined by experience, but recommended starting

values are 2.0 for the constant coefficient model, and 0.0 for the nonlinear model. The default values are all 2.0.

Time Difference Centering Parameters

The following parameters specify the type of time differencing scheme to be used. The generalized Beam and Warming (1978) time differencing formula is given by equation (3.1) of Volume 1.

THC	A 2-element array specifying the time differencing centering parameters θ_1 and θ_2 to be used for the continuity equation. The default values are 1.0 and 0.0.
THX	A 3-element array specifying the time differencing centering parameters θ_1 , θ_2 , and θ_3 to be used for the x-momentum equation. The default values are 1.0, 0.0, and 0.0.
THY	A 3-element array specifying the time differencing centering parameters θ_1 , θ_2 , and θ_3 to be used for the y-momentum equation. The default values are 1.0, 0.0, and 0.0.
THZ	A 3-element array specifying the time differencing centering parameters θ_1 , θ_2 , and θ_3 to be used for the swirl momentum equation. The default values are 1.0, 0.0, and 0.0.
THE	A 3-element array specifying the time differencing centering parameters θ_1 , θ_2 , and θ_3 to be used for the energy equation. The default values are 1.0, 0.0, and 0.0.
THKE	A 2-element array specifying the time differencing centering parameters θ_1 and θ_2 to be used for the k - ϵ equations. The default values are 1.0 and 0.0.

The following table summarizes the time differencing schemes that may be used. The Euler implicit method is recommended for steady flows, and the 3-point backward implicit method is recommended for unsteady flows.

θ_1	θ_2	θ_3	Method	Accuracy
1	0	0	Euler implicit	$O(\Delta\tau)$
1/2	0	1/2	Trapezoidal implicit	$O(\Delta\tau)^2$
1	1/2	1	3-point backward implicit	$O(\Delta\tau)^2$

3.1.9 Namelist TIME

Time Step Selection Parameters

The following parameters determine the procedure used to set the time step size for the mean flow equations, and to change it as the solution proceeds. The various options for IDTAU are described in more detail in the description of subroutine TIMSTP in Section 4.2 of Volume 3.

IDTMOD	The time step size $\Delta\tau$ is recomputed every IDTMOD'th step. The default value is 1.
IDTAU	1 for a <i>global</i> (i.e., constant in space) time step $\Delta\tau = (\text{CFL})\Delta\tau_{eff}$, where $\Delta\tau_{eff}$ is the minimum of the allowable time steps at each grid point based on the CFL criteria for explicit methods.

¹⁰ In ΔQ_{max} , the total energy \bar{E}_T has been divided by $E_{T,r} = \rho_r \bar{R} T_r / (\gamma_r - 1) + \rho_r u^2 / 2$ so that it is the same order of magnitude as the other conservation variables.

- 2 for a global time step initially computed using the IDTAU = 1 option, but adjusted as the solution proceeds based on ΔQ_{max} , the absolute value of the maximum change in the dependent variables.¹⁰ For any of the dependent variables, if $\Delta Q_{max} < CHG1$, the CFL number is multiplied by DTF1. If $\Delta Q_{max} > CHG2$, the CFL number is divided by DTF2. If $\Delta Q_{max} > 0.15$, the CFL number is cut in half. The CFL number will not be decreased below CFLMIN, or increased above CFLMAX.
- 3 for a global time step $\Delta\tau$ equal to the specified input DT.
- 4 for a global time step initially equal to the specified input DT, but adjusted as the solution proceeds based on ΔQ_{max} , the absolute value of the maximum change in the dependent variables.¹⁰ For any of the dependent variables, if $\Delta Q_{max} < CHG1$, $\Delta\tau$ is multiplied by DTF1. If $\Delta Q_{max} > CHG2$, $\Delta\tau$ is divided by DTF2. If $\Delta Q_{max} > 0.15$, $\Delta\tau$ is cut in half. $\Delta\tau$ will not be decreased below DTMIN, or increased above DTMAX.
- 5 for a *local* (i.e., varying in space) time step $\Delta\tau = (CFL)\Delta\tau_{eff}$, where $\Delta\tau_{eff}$ is the allowable time step at each grid point based on the CFL criteria for explicit methods.
- 6 for a local time step initially computed using the IDTAU = 5 option, but adjusted as the solution proceeds based on ΔQ_{max} , the absolute value of the maximum change in the dependent variables.¹⁰ For any of the dependent variables, if $\Delta Q_{max} < CHG1$, the CFL number is multiplied by DTF1. If $\Delta Q_{max} > CHG2$, the CFL number is divided by DTF2. If $\Delta Q_{max} > 0.15$, the CFL number is cut in half. The CFL number will not be decreased below CFLMIN, or increased above CFLMAX.
- 7 for a global time step with cycling. $\Delta\tau$ will be cycled repeatedly between DTMIN and DTMAX using a logarithmic progression over NDTCYC time steps. For some problems this option has been shown to dramatically speed convergence. However, the choice of DTMIN, DTMAX, and NDTCYC is critical, and no method has been developed that assures a good choice. Poor choices may even slow down convergence, so this option should be used with caution.
- 8 for a local time step computed using the procedure of Knight and Choi (1989). With this option, $\Delta\tau = \max[\Delta\tau_0, (\Delta\tau_{eff})_\xi]$, where $\Delta\tau_0 = (CFL) \min[(\Delta\tau_{eff})_\xi, (\Delta\tau_{eff})_\eta]$. The parameters $(\Delta\tau_{eff})_\xi$ and $(\Delta\tau_{eff})_\eta$ are the allowable time steps at each grid point based on the CFL criteria for explicit methods, computed separately for each computational coordinate direction. This formulation assumes that flow is generally in the ξ direction.
- 9 for a local time step computed as in the IDTAU = 8 option, but with a viscous correction added to the definitions of $(\Delta\tau_{eff})_\xi$ and $(\Delta\tau_{eff})_\eta$, similar to that used by Cooper (1987).

If IDTAU = 7, ICHECK and IDTMOD are both automatically set equal to 1, and NITAVG is set equal to NDTCYC. In addition, if IDTAU = 7 and ICTEST = 1, ICTEST is changed to 2. If IDTAU = 2, 4, or 6, IDTMOD is automatically set equal to ICHECK. The default value is 5.

The above parameters IDTAU and IDTMOD apply to every case. Which of the remaining parameters are needed depends on the value of IDTAU, as specified in the following table.

IDTAU Parameters Needed

1	CFL
2	CFL, CHG1, CHG2, DTF1, DTF2, CFLMIN, CFLMAX
3	DT
4	DT, CHG1, CHG2, DTF1, DTF2, DTMIN, DTMAX
5	CFL
6	CFL, CHG1, CHG2, DTF1, DTF2, CFLMIN, CFLMAX
7	DTMIN, DTMAX, NDTCYC
8	CFL
9	CFL

CFL	An array, given as CFL(ITSEQ), specifying the ratio $\Delta\tau/\Delta\tau_{eff}$, where $\Delta\tau$ is the actual time step used in the implicit calculation and $\Delta\tau_{eff}$ is the allowable time step based on the CFL criteria for explicit methods. The subscript ITSEQ is the sequence number. For time steps 1 through NTIME(1), CFL(1) will be used. Then for steps NTIME(1) + 1 through NTIME(1) + NTIME(2), CFL(2) will be used, etc. ¹¹ CFL is only used if IDTAU = 1, 2, 5, 6, 8, or 9. The default values are all 1.0.
DT	An array, given as DT(ITSEQ), specifying the time step size $\Delta\tau$. The subscript ITSEQ is the sequence number. For time steps 1 through NTIME(1), DT(1) will be used. Then for steps NTIME(1) + 1 through NTIME(1) + NTIME(2), DT(2) will be used, etc. ¹² DT is only used if IDTAU = 3 or 4. The default values are all 0.01.
CHG1	Minimum change, in absolute value, that is allowed in any dependent variable before increasing $\Delta\tau$. CHG1 is only used if IDTAU = 2, 4, or 6. The default value is 0.04.
CHG2	Maximum change, in absolute value, that is allowed in any dependent variable before decreasing $\Delta\tau$. CHG2 is only used if IDTAU = 2, 4, or 6. The default value is 0.06.
DTF1	Factor by which $\Delta\tau$ is multiplied if the solution changes too slowly. DTF1 is only used if IDTAU = 2, 4, or 6. The default value is 1.25.
DTF2	Factor by which $\Delta\tau$ is divided if the solution changes too quickly. DTF2 is only used if IDTAU = 2, 4, or 6. The default value is 1.25.
CFLMIN	Minimum value that the CFL number is allowed to reach. CFLMIN is only used if IDTAU = 2 or 6. The default value is 0.5.
CFLMAX	Maximum value that the CFL number is allowed to reach. CFLMAX is only used if IDTAU = 2 or 6. The default value is 10.0.
DTMIN	Minimum value that $\Delta\tau$ is allowed to reach (IDTAU = 4), or the minimum $\Delta\tau$ in the time step cycling procedure (IDTAU = 7.) The default value is 0.1.
DTMAX	Maximum value that $\Delta\tau$ is allowed to reach (IDTAU = 4), or the maximum $\Delta\tau$ in the time step cycling procedure (IDTAU = 7.) The default value is 0.1.

¹¹ Note that if IDTAU = 2 or 6, CFL(1) only sets $\Delta\tau$ for the first time step, and that the time step sequencing option does not apply.

¹² Note that if IDTAU = 4, DT(1) only sets $\Delta\tau$ for the first time step, and that the time step sequencing option does not apply.

NDTCYC Number of time steps per time step cycle. NDTCYC is used only with IDTAU = 7. The default value is 2, which results in a constant $\Delta\tau$ if DTMIN = DTMAX.

Time Marching Limits

These parameters determine the maximum number of time steps that will be taken.

NTSEQ The number of time step sequences being used. The maximum value allowed is the value of the dimensioning parameter NTSEQP. If NTSEQ > 1, IDTAU must be equal to 1, 3, or 5. (See Section 6.2.) The default value is 1.

NTIME An array, given as NTIME(ITSEQ), specifying the maximum number of time steps to march. The subscript ITSEQ varies from 1 to NTSEQ, and allows a series of different time steps to be specified by the values of CFL or DT. NTIME(ITSEQ) specifies the number of time steps within sequence ITSEQ. If NTSEQ = 3, for example, the total number of time steps taken will be $N_{total} = NTIME(1) + NTIME(2) + NTIME(3)$. The initial time level is level 1, and the final computed time level will be level $N_{total} + 1$. The default values are 10, 9*0.

TLIM When the amount of CPU time remaining for the job drops below TLIM seconds, the calculation is stopped. The default value is 20.0.

Convergence Testing Parameters

These parameters determine the convergence criteria to be used.

ICHECK Results are checked for convergence every ICHECK'th time level. The default value is 10.

ICTEST 1 to determine convergence based on the maximum change in absolute value of each of the conservation variables over a single time step, ΔQ_{max} .¹³
 2 to determine convergence based on the maximum change in absolute value of each of the conservation variables, averaged over the last NITAVG time steps, ΔQ_{avg} .¹³
 3 to determine convergence based on R_{L2} , the L_2 norm of the residual for each equation.
 4 to determine convergence based on R_{avg} , the average absolute value of the residual for each equation.
 5 to determine convergence based on R_{max} , the maximum absolute value of the residual for each equation.

Convergence is assumed when the maximum change or residual parameter is less than EPS. Note that the change in conservation variables over a time step is directly related to the size of the time step. Small time steps naturally yield small changes in conservation variables. With ICTEST = 1 or 2, therefore, convergence may be indicated prematurely.

If ICTEST = 2, ICHECK and IDTMOD are automatically set equal to 1. The default value is 3.

EPS Level of convergence to be reached, specified as EPS(IVAR) where IVAR varies from 1 to NEQ, corresponding to each conservation variable or equation. The default values are all 0.001.

¹³ The total energy \bar{E}_T is divided by $E_{Tr} = \rho_r \bar{R}T_r / (\gamma_r - 1) + \rho_r u_r^2 / 2$ before testing for convergence, so that it is the same order of magnitude as the other conservation variables.

NITAVG Number of time steps over which the maximum change in conservation variables is averaged to determine convergence. The maximum value allowed is the value of the dimensioning parameter NAMAX. (See Section 6.2.) NITAVG only applies to the ICTEST = 2 option. The default value is 10.

3.1.10 Namelist TURB

Control Parameters

The following parameters determine the type of turbulence model that will be used, and where it will be applied. These parameters apply to both the Baldwin-Lomax algebraic model and the Chien two-equation model.

ITURB 0 for laminar flow.
 1 for turbulent flow, using the algebraic eddy viscosity model of Baldwin and Lomax (1978), as described in Section 9.0 of Volume 1.
 20 for turbulent flow, using the two-equation $k-\epsilon$ model of Chien (1982), as described in Section 9.0 of Volume 1.

The default value is 0.

PRT If $PRT > 0.0$, it specifies the turbulent Prandtl number, which will be treated as constant. If $PRT \leq 0.0$, the turbulent Prandtl number will vary, and be computed using the empirical formula of Wassel and Catton (1973). The default value is 0.91.

LWALL1 A two-dimensional array, given as LWALL1(I2,IBOUND), specifying which points on the ξ boundaries are on a solid wall. Here I2 = 1 to N2 corresponding to each grid point on the boundary, and IBOUND = 1 or 2, corresponding to the $\xi = 0$ and $\xi = 1$ boundaries, respectively. Valid values of LWALL1(I2,IBOUND) are:

0 if the I2 point on boundary IBOUND is not on a solid wall.
 1 if the I2 point on boundary IBOUND is on a solid wall.

If LWALL1 is not specified, wall points on the ξ boundaries are identified by looking for points with zero velocity. Therefore, LWALL1 need only be specified for cases with non-zero velocity at the wall (e.g., a moving wall, bleed, blowing, etc.). LWALL1 is not needed if the boundary condition for boundary IBOUND is set using the KBC1(IBOUND) meta flag. The default values are all 0.

LWALL2 A two-dimensional array, given as LWALL2(I1,IBOUND), specifying which points on the η boundaries are on a solid wall. Here I1 = 1 to N1 corresponding to each grid point on the boundary, and IBOUND = 1 or 2, corresponding to the $\eta = 0$ and $\eta = 1$ boundaries, respectively. Valid values of LWALL2(I1,IBOUND) are:

0 if the I1 point on boundary IBOUND is not on a solid wall.
 1 if the I1 point on boundary IBOUND is on a solid wall.

If LWALL2 is not specified, wall points on the η boundaries are identified by looking for points with zero velocity. Therefore, LWALL2 need only be specified for cases with non-zero velocity at the wall (e.g., a moving wall, bleed, blowing, etc.). LWALL2 is not needed if the boundary condition for boundary IBOUND is set using the KBC2(IBOUND) meta flag. The default values are all 0.

- IWALL1** A 2-element array, specified as IWALL1(IBOUND), specifying which ξ boundaries are solid walls.¹⁴ The subscript IBOUND = 1 or 2, corresponding to the $\xi = 0$ and $\xi = 1$ boundaries, respectively. Valid values of IWALL1(IBOUND) are:
- 0 if the boundary is not a solid wall.
 - 1 if the boundary is a solid wall.
- IWALL1(IBOUND) is not needed if the boundary condition for boundary IBOUND is set using the KBC1(IBOUND) meta flag. The default values are both - 1.
- IWALL2** A 2-element array, specified as IWALL2(IBOUND), specifying which η boundaries are solid walls. The subscript IBOUND = 1 or 2, corresponding to the $\eta = 0$ and $\eta = 1$ boundaries, respectively. Valid values of IWALL2(IBOUND) are:
- 0 if the boundary is not a solid wall.
 - 1 if the boundary is a solid wall.
- IWALL2(IBOUND) is not needed if the boundary condition for boundary IBOUND is set using the KBC2(IBOUND) meta flag. The default values are both - 1.

Baldwin-Lomax Turbulence Model Parameters

These parameters apply only to the Baldwin-Lomax algebraic eddy viscosity model. Note, however, that they are used when the Baldwin-Lomax model is used to generate initial turbulent viscosity values for the Chien k - ϵ model (i.e., when IREST = 0, 1, or 3.)

The following two parameters specify which directions are important in computing the turbulent viscosity coefficient.

- ITXI** 0 to bypass computation of turbulent viscosity on lines in the ξ direction.
 1 to compute turbulent viscosity on lines in the ξ direction (i.e., due to walls at $\eta = 0$ and/or $\eta = 1$, or due to a free turbulent flow in the ξ direction.)
- If ITHIN(1) = 1, ITXI is automatically set equal to 1. The default value is 1.
- ITETA** 0 to bypass computation of turbulent viscosity on lines in the η direction.
 1 to compute turbulent viscosity on lines in the η direction (i.e., due to walls at $\xi = 0$ and/or $\xi = 1$, or due to a free turbulent flow in the η direction.)

If ITHIN(2) = 1, ITETA is automatically set equal to 1. The default value is 0.

The following two options may be used to modify the Baldwin-Lomax model.

- INNER** 1 to use the inner layer model of Baldwin and Lomax (1978).
 2 to use the inner layer model of Spalding (1961) and Kleinstein (1967).

The default value is 1.

- ILDAMP** 0 to use the normal Baldwin-Lomax mixing length formula in the inner region.
 1 to use the modified mixing length formula of Launder and Priddin (1973) in the inner region of the Baldwin-Lomax model.

¹⁴ The IWALL parameters have been superseded by the LWall parameters. However, for compatibility with previous versions of *Proteus*, the IWALL parameters have been left in, and if set equal to 0 or 1 will override the corresponding LWall values.

The default value is 0.

The following two parameters are used in the laminar-turbulent transition model of Cebeci and Bradshaw (1984).

- REXT1 The Reynolds number at the beginning of the transition region. This parameter only applies to cases with flow predominantly in the ξ direction, and with a leading edge at $\xi = 0$. The Reynolds number is based on maximum total velocity and distance from $\xi = 0$. The default value is 0.0.
- REXT2 The Reynolds number at the beginning of the transition region. This parameter only applies to cases with flow predominantly in the η direction, and with a leading edge at $\eta = 0$. The Reynolds number is based on maximum total velocity and distance from $\eta = 0$. The default value is 0.0.

The following parameters are various constants used in the Baldwin-Lomax model.

- CCLAU The Clauser constant K used in the Baldwin-Lomax outer region model. The default value is 0.0168.
- CCP The constant C_p used in the Baldwin-Lomax outer region model. The default value is 1.6.
- CWK The constant C_{wk} used in the formula for F_{wake} in the Baldwin-Lomax outer region model. The default value is 0.25.
- CKLEB The constant C_{Kleb} used in the formula for the Klebanoff intermittency factor F_{Kleb} in the Baldwin-Lomax outer region model. The default value is 0.3.
- CKMIN The constant $(C_{Kleb})_{min}$ used in the formula for the Klebanoff intermittency factor F_{Kleb} in the Baldwin-Lomax outer region model. The default value is normally 0.0. However, when the Baldwin-Lomax model is being used to generate initial turbulent viscosity values for the Chien $k-\epsilon$ model (ITURB = 20 and IREST = 0, 1, or 3), the default value is 0.1.
- APLUS The Van Driest damping constant A^+ used in the Baldwin-Lomax outer and inner region models. The default value is 26.0.
- CB The constant B used in the formula for the Klebanoff intermittency factor F_{Kleb} in the Baldwin-Lomax outer region model, and in the Spalding-Kleinstein inner region model. The default value is 5.5.
- CVK The Von Karman mixing length constant κ used in both the Baldwin-Lomax and Spalding-Kleinstein inner region models. The default value is 0.4.
- CNL The exponent n in the Launder-Priddin modified mixing length formula. The default value is 1.7.
- CNA The exponent n in the formula used to average the two outer region μ_t profiles that result when both boundaries in a coordinate direction are solid surfaces. The default value is 2.0.

Chien Turbulence Model Parameters

These parameters apply only to the Chien $k-\epsilon$ model. The following two parameters specify the time step size for the $k-\epsilon$ equations.

NTKE	Number of k - ε time iterations per mean flow time iteration. Note that this must be an integer equal to or greater than 1. Past experience indicates that this value should generally be less than 50. The default value is 1.
TFACT	Factor used in computing the k - ε time step. Note that this can be any real positive number. The time step size for the k - ε equations will be equal to $\Delta\tau_{k-\varepsilon} = \text{TFACT}(\Delta\tau)$, where $\Delta\tau$ is the time step size for the mean flow equations. For equal k - ε and mean flow time steps, use $\text{TFACT} = 1/\text{NTKE}$. The default value is 1.0.

The following parameters are various constants used in the Chien k - ε model.

CMUR	Constant C_{μ_r} used to compute C_μ in the turbulent viscosity formula for the k - ε equations. The default value is 0.09.
CONE	Constant C_1 used in the production term of the ε equation. The default value is 1.35.
CTHREE	Constant C_3 used to compute C_μ in the turbulent viscosity formula for the k - ε equations. The default value is 0.0115.
CTWOR	Constant C_2 used to compute C_2 in the dissipation term of the ε equation. The default value is 1.8.
SIGE	The constant σ_ε used in the diffusion term of the ε equation. The default value is 1.3.
SIGK	The constant σ_k used in the diffusion term of the k equation. The default value is 1.0.

3.1.11 Namelist IC

This namelist is used in subroutine INIT to read in parameters needed in setting up the initial conditions for the mean flow. The version of INIT built into *Proteus* specifies uniform flow with constant properties everywhere in the flow field. In general, however, the user will supply a version of INIT tailored to the problem being solved. This namelist, then, may be modified by the user to read in parameters different from those listed here.

P0	Initial static pressure p_0 . The default value is 1.0.
T0	Initial static temperature T_0 . The default value is 1.0.
U0	Initial x-direction velocity u_0 . The default value is 0.0.
V0	Initial y-direction velocity v_0 . The default value is 0.0.
W0	Initial swirl velocity w_0 . The default value is 0.0.

3.2 COORDINATE SYSTEM FILE

The type of computational coordinate system to be used is controlled by the input parameter NGEOM in namelist GMTRY. For $\text{NGEOM} = 10$, the coordinate system is read from a pre-stored file. This file may be created by any body-fitted coordinate system generator available to the user. The coordinates may be nonorthogonal.

The metric coefficients and Jacobian describing the nonorthogonal grid transformation are computed internally by *Proteus*. This calculation involves numerically computing first derivatives of the user-specified coordinates. Since *Proteus* solves the Navier-Stokes equations in fully conservative form, the metric coefficients themselves are factors in terms whose first and second derivatives are also computed numerically.

In effect, then, third derivatives of the user-specified coordinates are used in the solution. Care should therefore be taken in ensuring that these coordinates are smooth. No coordinate smoothing is done by *Proteus* itself.

The Cartesian (x,y) or cylindrical (x,r) coordinates describing the computational coordinate system are read from an unformatted file as follows:

```

      read (ngrid) ng1,ng2
      read (ngrid) ((xc(j1,j2),j1=1,ng1),j2=1,ng2),
&                ((yc(j1,j2),j1=1,ng1),j2=1,ng2)

```

The parameters read from the file are defined as follows:

NG1	Number of points in the ξ direction. The maximum value allowed is the value of the dimensioning parameter N1P. (See Section 6.2.)
NG2	Number of points in the η direction. The maximum value allowed is the value of the dimensioning parameter N2P. (See Section 6.2.)
XC	Cartesian or cylindrical x -coordinate.
YC	Cartesian or cylindrical y or r -coordinate.

Note that the number of points NG1 and NG2 used to specify the computational *coordinate system* need not be the same as the number of points N1 and N2 used in the computational *mesh*. The coordinates of the points in the computational mesh, which is the mesh used in the *Proteus* solution, will be found by interpolation among the points in the computational coordinate system.

TABLE 3-1. - NORMALIZING CONDITIONS

Variable	Normalizing Value
Length	$L_n = L_r$
Velocity	$u_n = u_r$
Temperature	$T_n = T_r$
Density	$\rho_n = \rho_r$
Viscosity	$\mu_n = \mu_r$
Thermal conductivity	$k_n = k_r$
Pressure	$p_n = \rho_r u_r^2$
Energy per unit volume	$e_n = \rho_r u_r^2$
Gas constant	$R_n = u_r^2 / T_r$
Specific heat	$c_{p,n} = u_r^2 / T_r$
Enthalpy	$h_n = u_r^2$
Time	$t_n = L_r / u_r$
Turbulent kinetic energy	$k_n = u_r^2$
Turbulent dissipation rate	$\varepsilon_n = \rho_r u_r^4 / \mu_r$

TABLE 3-2. - REFERENCE CONDITIONS

Variable	Reference Value
Length	L_r
Velocity	u_r
Temperature	T_r
Density	ρ_r
Viscosity	μ_r
Thermal conductivity	k_r
Pressure	$p_r = \rho_r \bar{R} T_r / g_c$
Energy per unit volume	$e_r = \rho_r u_r^2$
Enthalpy	u_r^2
Specific heat	u_r^2 / T_r
Time	L_r / u_r
Turbulent kinetic energy	u_r^2
Turbulent dissipation rate	$\rho_r u_r^4 / \mu_r$

TABLE 3-3. - OUTPUT VARIABLES

IVOUT	VARIABLE	DEFINITION
Velocities		
1	x-velocity	u
2	y or r-velocity	v
3	Swirl velocity	w
4	Mach number	$M = \frac{ \vec{V} }{a}$
5	Speed of sound	$a = \sqrt{\gamma RT}$
6	Contravariant velocity normal to ξ surface	$(\xi_t + u\xi_x + v\xi_y)/(\xi_x^2 + \xi_y^2)^{1/2}$
7	Contravariant velocity normal to η surface	$(\eta_t + u\eta_x + v\eta_y)/(\eta_x^2 + \eta_y^2)^{1/2}$
8		
9	Total velocity magnitude	$ \vec{V} = (u^2 + v^2 + w^2)^{1/2}$
10	x-momentum	ρu
11	y or r-momentum	ρv
12	Swirl momentum	ρw
13	ξ -velocity	$V_\xi = (\eta_y u - \eta_x v)/(\eta_x^2 + \eta_y^2)^{1/2}$
14	η -velocity	$V_\eta = (-\xi_y u + \xi_x v)/(\xi_x^2 + \xi_y^2)^{1/2}$
15	Not used	
16	Flow angle α_v , deg.	$\tan^{-1} \frac{v}{u}$
17	Flow angle α_w , deg.	$\tan^{-1} \frac{w}{u}$
Densities		
20	Static density	ρ
21	Total density	$\rho_T = \rho \left(1 + \frac{\gamma - 1}{2} M^2 \right)^{1/(\gamma - 1)}$

IVOUT	VARIABLE	DEFINITION
Pressures		
30	Static pressure	p
31	Total pressure	$p_T = p \left(1 + \frac{\gamma - 1}{2} M^2 \right)^{\gamma/(\gamma - 1)}$
32	Static pressure coefficient	$c_p = \frac{\bar{p} - p_r}{\rho_r \mu_r^2 / 2g_c}$
33	Total pressure coefficient	$c_{p_T} = \frac{\bar{p}_T - p_{T_r}}{\rho_r \mu_r^2 / 2g_c}$
34	Pitot pressure	$p_p = p_T \quad \text{if } M \leq 1$ $p_p = p \left(\frac{\gamma + 1}{2} M^2 \right)^{\gamma/(\gamma - 1)}$ $\left(\frac{2\gamma}{\gamma + 1} M^2 - \frac{\gamma - 1}{\gamma + 1} \right)^{-1/(\gamma - 1)} \quad \text{if } M > 1$
35	Dynamic pressure	$\frac{1}{2} \rho (u^2 + v^2 + w^2) \frac{\rho_r \mu_r^2}{g_c p_r}$
Temperatures		
40	Static temperature	T
41	Total temperature	$T_T = T \left(1 + \frac{\gamma - 1}{2} M^2 \right)$
Energies		
50	Total energy per unit volume	E_T
51	Total energy	$\frac{E_T}{\rho}$
52	Internal energy	$e_i = c_v T$
53	Kinetic energy	$e_k = \frac{1}{2} (u^2 + v^2 + w^2)$
Enthalpies		
60	Static enthalpy	$h = c_p T$
61	Total enthalpy	$h_T = c_p T_T$

IVOUT	VARIABLE	DEFINITION
Vorticities		
70	x-vorticity	$\Omega_x = \frac{\partial w}{\partial y} \quad \left(+ \frac{w}{y} \text{ if axisymmetric} \right)$
71	y or r-vorticity	$\Omega_y = - \frac{\partial w}{\partial x}$
72	z or θ -vorticity	$\Omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$
73	Total vorticity magnitude	$ \vec{\Omega} = (\Omega_x^2 + \Omega_y^2 + \Omega_z^2)^{1/2}$
Entropies		
80	Entropy	$s = \bar{c}_v \ln \left(\frac{\bar{p}}{p_r} \right) + \bar{c}_p \ln \left(\frac{\rho_r}{\rho} \right)$
Temperature-Dependent Parameters		
90	Laminar viscosity coefficient	$\mu_l = \mu - \mu_t$
91	Laminar second coefficient of viscosity	$\lambda_l = - \frac{2\mu_l}{3}$
92	Laminar thermal conductivity coefficient	$k_l = k - k_t$
93	Specific heat at constant pressure	\bar{c}_p
94	Specific heat at constant volume	\bar{c}_v
95	Ratio of specific heats	$\gamma = \frac{c_p}{c_v}$

IVOUT	VARIABLE	DEFINITION
Turbulence Parameters		
100	Turbulent viscosity coefficient	μ_t
101	Turbulent second coefficient of viscosity	$\lambda_t = -\frac{2\mu_t}{3}$
102	Turbulent thermal conductivity coefficient	$k_t = \frac{\bar{c}_p \bar{\mu}_t}{Pr_t} \frac{1}{k_r}$
103	Effective viscosity coefficient	μ
104	Effective second coefficient of viscosity	λ
105	Effective thermal conductivity coefficient	k
106	Turbulent kinetic energy	k
107	Turbulent dissipation rate	ε
108	Inner region coordinate	$y^+ = \frac{\rho_w u_z y_n}{\mu_w} Re_r$
109	Inner region velocity	$u^+ = \frac{ \vec{V} }{u_\tau} = \vec{V} \left(\frac{1}{Re_r} \frac{\mu}{\rho} \vec{\Omega} \right)^{-1/2}_w$
Gradients		
120	Shear stress	τ_{xx}
121	Shear stress	τ_{yy} OR τ_{rr}
122	Shear stress	$\tau_{\theta\theta}$
123	Shear stress	τ_{xy} OR τ_{xr}
124	Shear stress	$\tau_{x\theta}$
125	Shear stress	$\tau_{r\theta}$
126	Heat flux	q_x
127	Heat flux	q_y OR q_r

IVOUT	VARIABLE	DEFINITION
Coordinates		
200	Cartesian x -coordinate	x
201	Cartesian or cylindrical y or r -coordinate	y or r
202	Not used	
203	Local Δx	$ \mathcal{J}^{-1} (\eta_y \Delta \xi - \xi_y \Delta \eta)$
204	Local Δy	$ \mathcal{J}^{-1} (-\eta_x \Delta \xi + \xi_x \Delta \eta)$
205	Not used	
206	Local cell Reynolds number Re_c	$Re_r \frac{\rho \vec{V} [(\Delta x)^2 + (\Delta y)^2]^{1/2}}{\mu}$
207	Local x -direction cell Reynolds number $(Re_c)_x$	$Re_r \frac{\rho u \Delta x}{\mu}$
208	Local y -direction cell Reynolds number $(Re_c)_y$	$Re_r \frac{\rho v \Delta y}{\mu}$
209	Not used	
Metric Parameters		
210	Inverse of the grid transformation Jacobian	\mathcal{J}^{-1}
211	Metric coefficient	ξ_t
212	Metric coefficient	ξ_x
213	Metric coefficient	ξ_y
214	Not used	
215	Metric coefficient	η_t
216	Metric coefficient	η_x
217	Metric coefficient	η_y
218	Not used	
219	Not used	
220	Not used	
221	Not used	
222	Not used	

IVOUT	VARIABLE	DEFINITION
Times		
230	Time step size	$\Delta\tau$
231	Time	τ
232	Local CFL number	$\Delta\tau \left\{ \frac{ \xi_t + u\xi_x + v\xi_y }{\Delta\xi} + \frac{ \eta_t + u\eta_x + v\eta_y }{\Delta\eta} + a \left[\left(\frac{\xi_x}{\Delta\xi} + \frac{\eta_x}{\Delta\eta} \right)^2 + \left(\frac{\xi_y}{\Delta\xi} + \frac{\eta_y}{\Delta\eta} \right)^2 \right]^{1/2} \right\}$
233	Local ξ -direction CFL number	$\Delta\tau \left[\frac{ \xi_t + u\xi_x + v\xi_y }{\Delta\xi} + a \frac{(\xi_x^2 + \xi_y^2)^{1/2}}{\Delta\xi} \right]$
234	Local η -direction CFL number	$\Delta\tau \left[\frac{ \eta_t + u\eta_x + v\eta_y }{\Delta\eta} + a \frac{(\eta_x^2 + \eta_y^2)^{1/2}}{\Delta\eta} \right]$
235	Not used	

TABLE 3-4. - EQUATIONS SOLVED

IHSTAG	ISWIRL	NEQ	Order of Equations	Order of Dependent Variables
0	0	4	Continuity, x-momentum, y or r-momentum, energy	$\rho, \rho u, \rho v, E_T$
1	0	3	Continuity, x-momentum, y or r-momentum	$\rho, \rho u, \rho v$
1	1	4	Continuity, x-momentum, y or r-momentum, swirl momentum	$\rho, \rho u, \rho v, \rho w$
0	1	5	Continuity, x-momentum, y or r-momentum, swirl momentum, energy	$\rho, \rho u, \rho v, \rho w, E_T$

TABLE 3-5. - BOUNDARY TYPES

KBC VALUE ^a	BOUNDARY TYPE	JBC VALUES SET	EQUATIONS
± 1	No-slip adiabatic wall	11, 21, 31, ± 43 , ± 53	$u = v = w = 0, \partial p / \partial n = \partial T / \partial n = 0$
± 2	No-slip wall, specified temperature	11, 21, 31, ± 43 , 50	$u = v = w = 0, \partial p / \partial n = 0, \Delta T = 0$
± 3	Inviscid wall	± 33 , ± 43 , ± 53 , 71, 79	$\partial w / \partial n = \partial p / \partial n = \partial T / \partial n = 0$, $V_n = 0, \partial^2 V_i / \partial \phi^2 = 0$
10	Subsonic inflow, linear extrapolation	14, 24, 34, 46, 56	$\partial^2 u / \partial \phi^2 = \partial^2 v / \partial \phi^2 = \partial^2 w / \partial \phi^2 = 0$, $\Delta p_T = \Delta T_T = 0$
± 11	Subsonic inflow, zero gradient	± 12 , ± 22 , ± 32 , 46, 56	$\partial u / \partial \phi = \partial v / \partial \phi = \partial w / \partial \phi = 0$, $\Delta p_T = \Delta T_T = 0$
20	Subsonic outflow, linear extrapolation	14, 24, 34, 40, 54	$\partial^2 u / \partial \phi^2 = \partial^2 v / \partial \phi^2 = \partial^2 w / \partial \phi^2 = 0$, $\Delta p = 0, \partial^2 T / \partial \phi^2 = 0$
± 21	Subsonic outflow, zero gradient	± 12 , ± 22 , ± 32 , 40, ± 52	$\partial u / \partial \phi = \partial v / \partial \phi = \partial w / \partial \phi = 0$, $\Delta p = 0, \partial T / \partial \phi = 0$
30	Supersonic inflow	10, 20, 30, 40, 50	$\Delta u = \Delta v = \Delta w = \Delta p = \Delta T = 0$
40	Supersonic outflow, linear extrapolation	14, 24, 34, 44, 54	$\partial^2 u / \partial \phi^2 = \partial^2 v / \partial \phi^2 = \partial^2 w / \partial \phi^2 = 0$, $\partial^2 p / \partial \phi^2 = \partial^2 T / \partial \phi^2 = 0$
± 41	Supersonic outflow, zero gradient	± 12 , ± 22 , ± 32 , ± 42 , ± 52	$\partial u / \partial \phi = \partial v / \partial \phi = \partial w / \partial \phi = 0$, $\partial p / \partial \phi = \partial T / \partial \phi = 0$
± 50	Symmetry	± 33 , ± 43 , ± 53 , 71, ± 78	$\partial w / \partial n = \partial p / \partial n = \partial T / \partial n = 0$, $V_n = 0, \partial V_i / \partial n = 0$
60	Spatially periodic		$Q_1 = Q_{N_1}$ or $Q_1 = Q_{N_2}$

^a Use the "+" sign for 2-point one-sided differencing of first derivatives, and the "-" sign for 3-point differencing of first derivatives.

TABLE 3-6. - BOUNDARY CONDITION TYPES FOR MEAN FLOW

JBC OR IBC VALUE ^a	EQUATION	DESCRIPTION
Conservation Variable Boundary Conditions		
0	$\Delta Q = 0$	No change from initial conditions.
1	$Q = f$	Specified conservation variable.
± 2	$\partial Q / \partial \phi = f$	Specified coordinate direction gradient.
± 3	$\partial Q / \partial n = f$	Specified normal direction gradient ^b .
4	$\partial^2 Q / \partial \phi^2 = 0$	Linear extrapolation.
5		Not used.
6		Not used.
7		Not used.
8		Not used.
9		Not used.
x-Velocity Boundary Conditions		
10	$\Delta u = 0$	No change from initial conditions.
11	$u = f$	Specified x-velocity.
± 12	$\partial u / \partial \phi = f$	Specified coordinate direction gradient.
± 13	$\partial u / \partial n = f$	Specified normal direction gradient ^b .
14	$\partial^2 u / \partial \phi^2 = 0$	Linear extrapolation.
15		Not used.
16		Not used.
17		Not used.
18		Not used.
19		Not used.
y or r-Velocity Boundary Conditions		
20	$\Delta v = 0$	No change from initial conditions.
21	$v = f$	Specified y or r-velocity.
± 22	$\partial v / \partial \phi = f$	Specified coordinate direction gradient.
± 23	$\partial v / \partial n = f$	Specified normal direction gradient ^b .
24	$\partial^2 v / \partial \phi^2 = 0$	Linear extrapolation.
25		Not used.
26		Not used.
27		Not used.
28		Not used.
29	$\tan^{-1}(v/u) = f$	Specified flow angle in degrees.
Swirl Velocity Boundary Conditions		
30	$\Delta w = 0$	No change from initial conditions.
31	$w = f$	Specified swirl velocity.
± 32	$\partial w / \partial \phi = f$	Specified coordinate direction gradient.
± 33	$\partial w / \partial n = f$	Specified normal direction gradient ^b .
34	$\partial^2 w / \partial \phi^2 = 0$	Linear extrapolation.
35		Not used.
36		Not used.
37		Not used.
38		Not used.
39	$\tan^{-1}(w/u) = f$	Specified flow angle in degrees.

JBC OR IBC VALUE ^a	EQUATION	DESCRIPTION
Pressure Boundary Conditions		
40	$\Delta p = 0$	No change from initial conditions.
41	$p = f$	Specified static pressure.
± 42	$\partial p / \partial \phi = f$	Specified coordinate direction gradient.
± 43	$\partial p / \partial n = f$	Specified normal direction gradient ^b .
44	$\partial^2 p / \partial \phi^2 = 0$	Linear extrapolation.
45		Not used.
46	$\Delta p_T = 0$	No change from initial conditions.
47	$p_T = f$	Specified total pressure.
48		Not used.
49		Not used.
Temperature Boundary Conditions		
50	$\Delta T = 0$	No change from initial conditions.
51	$T = f$	Specified static temperature.
± 52	$\partial T / \partial \phi = f$	Specified coordinate direction gradient.
± 53	$\partial T / \partial n = f$	Specified normal direction gradient ^b .
54	$\partial^2 T / \partial \phi^2 = 0$	Linear extrapolation.
55		Not used.
56	$\Delta T_T = 0$	No change from initial conditions.
57	$T_T = f$	Specified total temperature.
58		Not used.
59		Not used.
Density Boundary Conditions		
60	$\Delta \rho = 0$	No change from initial conditions.
61	$\rho = f$	Specified static density.
± 62	$\partial \rho / \partial \phi = f$	Specified coordinate direction gradient.
± 63	$\partial \rho / \partial n = f$	Specified normal direction gradient ^b .
64	$\partial^2 \rho / \partial \phi^2 = 0$	Linear extrapolation.
65		Not used.
66		Not used.
67		Not used.
68		Not used.
69		Not used.
Normal ^c and Tangential Velocity Boundary Conditions		
70		Not used.
71	$V_n = f$	Specified normal velocity.
± 72	$\partial V_n / \partial \phi = f$	Specified coordinate direction gradient.
± 73	$\partial V_n / \partial n = f$	Specified normal direction gradient ^b .
74	$\partial^2 V_n / \partial \phi^2 = 0$	Linear extrapolation.
75		Not used.
76	$V_t = f$	Specified tangential velocity.
± 77	$\partial V_t / \partial \phi = f$	Specified coordinate direction gradient.
± 78	$\partial V_t / \partial n = f$	Specified normal direction gradient ^b .
79	$\partial^2 V_t / \partial \phi^2 = 0$	Linear extrapolation.

JBC OR IBC VALUE ^a	EQUATION	DESCRIPTION
User-Supplied Boundary Conditions		
90	$\Delta F = 0$	No change from initial conditions.
91	$F = f$	Specified function.
\pm 92	$\partial F / \partial \phi = f$	Specified coordinate direction gradient.
\pm 93	$\partial F / \partial n = f$	Specified normal direction gradient ^b .
94	$\partial^2 F / \partial \phi^2 = 0$	Linear extrapolation.
95		Not used.
96		Not used.
97		Not used.
98		Not used.
99		Not used.

^a Use the "+" sign for 2-point one-sided differencing, and the "-" sign for 3-point one-sided differencing.

^b Normal derivatives are positive in the direction of increasing ξ or η .

^c Normal velocity is positive in the direction of increasing ξ or η .

TABLE 3-7. - BOUNDARY CONDITION TYPES FOR k - ϵ EQUATIONS

JBCT OR IBCT VALUE ^a	EQUATION	DESCRIPTION
Turbulent Kinetic Energy Boundary Conditions		
0	$\Delta k = 0$	No change from initial conditions.
1	$k = f$	Specified turbulent kinetic energy.
± 2	$\partial k / \partial \phi = f$	Specified coordinate direction gradient.
3		
4	$\partial^2 k / \partial \phi^2 = 0$	Linear extrapolation.
5		
6		
7		
8		
9		
Turbulent Dissipation Rate Boundary Conditions		
10	$\Delta \epsilon = 0$	No change from initial conditions.
11	$\epsilon = f$	Specified turbulent dissipation rate.
± 12	$\partial \epsilon / \partial \phi = f$	Specified coordinate direction gradient.
13		
14	$\partial^2 \epsilon / \partial \phi^2 = 0$	Linear extrapolation.
15		
16		
17		
18		
19		

^a Use the "+" sign for 2-point one-sided differencing, and the "-" sign for 3-point one-sided differencing.

26 INTENTIONALLY BLANK

4.0 OUTPUT DESCRIPTION

Several output files may be created during a *Proteus* run. The standard output is a formatted file written to Fortran unit NOUT that is intended for printing. Additional unformatted files may be written for use as input by various post-processing programs. Unformatted restart files may also be written for use as input for a subsequent *Proteus* run.

4.1 STANDARD OUTPUT

The standard *Proteus* output is a formatted file written to Fortran unit NOUT, and is intended for printing. Actual examples of typical standard output files are presented in Section 9.0. Unless specified otherwise, all of the output parameters in the standard output are nondimensional, with the appropriate reference condition from Table 3-2 as the nondimensionalizing factor.

4.1.1 Title Page and Namelists

The standard *Proteus* output begins with a title page,¹⁵ which identifies the version of *Proteus* being run and lists the user-specified title for the run. This is followed by a printout of the contents of the input namelists RSTRT, IO, GMTRY, FLOW, BC, NUM, TIME, and TURB. Note that, for variables not specified by the user in the input namelists, the values in this printout will be the default values.

4.1.2 Normalizing and Reference Conditions

The dimensional values for the normalizing and reference conditions are printed on the next page, with the appropriate units as set by the input parameter IUNITS. The normalizing conditions are the parameters used to nondimensionalize the governing equations. The reference conditions are used during input and output for nondimensionalization of various parameters and for specifying various flow conditions. The distinction between normalizing and reference conditions is described in greater detail in Section 3.1.1. They are listed in Tables 3-1 and 3-2.

After the printout of the normalizing and reference parameters comes anything written to unit NOUT by the user-supplied subroutine INIT. For the default version of INIT supplied with *Proteus*, this consists only of the contents of namelist IC.

4.1.3 Boundary Conditions

The next page is a printout of the boundary conditions being used. The boundary condition parameters JBC and GBC are printed in a box-like manner, with the values for the $\xi = 0$ and $\xi = 1$ surfaces on the left and right, and the values for the $\eta = 0$ and $\eta = 1$ surfaces on the bottom and top. If the Chien two-equation turbulence model is being used, the boundary condition parameters for the k - ϵ equations, JBCT and GBCT, are printed next, in the same box-like manner. If time-dependent boundary conditions are being used, this is followed by a listing of the input tables of GTBC vs. NTBCA.

4.1.4 Computed Flow Field

The bulk of the standard *Proteus* output consists of printout of the computed flow field. The input array IVOUT determines which variables are printed, as described in Section 3.1.4. The variables currently available for printing are listed and defined in Table 3-3. The printout for each variable at a given time level will begin on a separate page. The header for each variable will include the time level n , and, for global time

¹⁵ In this discussion, when "pages" of output are referred to it is assumed that the file is printed with Fortran carriage control in effect.

steps (IDTAU = 1 - 4, 7), the time t and time increment Δt in seconds. In the flow field printout, each column corresponds to a ξ location, and each row to an η location. The columns and rows are numbered with the ξ and η indices.

Flow field results are printed at time levels and grid points specified by the user through parameters in namelist IO. Since this printout can be very lengthy, the user is encouraged to minimize the amount of printed output by making judicious use of these parameters. Usually, the computed results can be examined most efficiently using post-processing graphics routines like CONTOUR or PLOT3D. (See Section 4.2).

After the final flow field printout, if the run ends normally a message is printed indicating whether or not the calculation converged.

4.1.5 Boundary Parameters

After each flow field printout, various parameters may be printed along the boundaries. The input arrays IWOUT1 and IWOUT2 determine whether or not these parameters are printed, as described in Section 3.1.4. The parameters printed are defined below. Note that some of these are meaningful only if the boundary is a solid wall.

X, Y	Cartesian coordinates x and y .
P	Static pressure p .
CF	Skin friction coefficient c_f , defined as

$$c_f = \frac{\bar{\mu} \frac{\partial \bar{V}_t}{\partial n}}{\frac{1}{2} \rho_r \mu_r^2} = \frac{2}{Re_r} \mu \frac{\partial V_t}{\partial n}$$

where the overbar denotes a dimensional quantity. In this equation $\partial V_t / \partial n$ represents the normal derivative of the tangential velocity.

TAUW	Shear stress τ_w , defined as
------	------------------------------------

$$\tau_w = \mu \frac{\partial V_t}{\partial n}$$

τ_w is thus nondimensionalized by $\mu_r u_r / L_r$.

T	Static temperature T .
QW	Heat flux q_w , defined as

$$q_w = -k \frac{\partial T}{\partial n}$$

In this equation $\partial T / \partial n$ represents the normal derivative of the temperature. q_w is thus nondimensionalized by $k_r T_r / L_r$.

H	Heat transfer coefficient h , defined as
---	--

$$h = \frac{q_w}{T - 1} = \frac{-k \frac{\partial T}{\partial n}}{T - 1}$$

h is thus nondimensionalized by k_r / L_r .

ST	Stanton number St , defined as
----	----------------------------------

$$St = \frac{\bar{h}}{\rho_r \mu_r \bar{c}_p} = \frac{h}{c_p} \frac{1}{Re_r Pr_r}$$

The boundary parameters are printed at the same time levels as the flow field printout, but at every point on the boundary, not just those specified by the input parameters IPRT1 and IPRT2, or IPRT1A and IPRT2A.

For this printout, normal derivatives are computed using a normal vector \vec{n} directed into the flow field. This means, for example, that the skin friction c_f will be positive for attached flow, even on the upper boundary.

4.1.6 Convergence History

In evaluating the results of a steady *Proteus* calculation, it's important to consider the level of convergence. This may be done by examining one of the forms of the residual for each equation. The residual is simply the number resulting from evaluating the steady form of the equation at a specific grid point and time (or iteration) level. Ideally, the residuals would all approach zero at convergence. In practice, however, for "real" problems they often drop to a certain level and then level off. Continuing the calculation beyond this point will not improve the results.

A decrease in the L_2 norm of the residual of three orders of magnitude is sometimes considered sufficient. Convergence, however, is in the eye of the beholder. The amount of decrease in the residual necessary for convergence will vary from problem to problem. For some problems, it may even be more appropriate to measure convergence by some flow-related parameter, such as the lift coefficient for an airfoil. Determining when a solution is sufficiently converged is, in some respects, a skill best acquired through experience.

At the end of a *Proteus* calculation, if first-order time differencing and steady boundary conditions were used, a summary of the convergence history is printed for each governing equation.¹⁶ The parameters in this printout are defined as follows:

LEVEL	Time level n .
CHGMAX	Maximum change in absolute value of the dependent variables from time level $n - 1$ to n . ¹⁷

$$\Delta Q_{max} = \max |\Delta Q_{i,j}^{n-1}|$$

CHGAVG	Maximum change in absolute value of the dependent variables, averaged over the last NITAVG time steps. ¹⁷
--------	--

$$\Delta Q_{avg} = \frac{1}{NITAVG} \sum_{m=n-NITAVG}^n \Delta Q_{max}^{m-1}$$

RESL2	The L_2 norm of the residual at time level n .
-------	--

$$R_{L_2} = \left(\sum (R_{i,j}^n)^2 \right)^{1/2}$$

RESAVG	The average absolute value of the residual at time level n , R_{avg} .
RESMAX	The maximum absolute value of the residual at time level n , R_{max} .
LRMAX	The grid indices (i, j) corresponding to the location of R_{max} .

¹⁶ Second-order time differencing should be used only for unsteady problems, for which "convergence" has no meaning. It should also be noted that the computation of the residuals in the code is correct only for first-order time differencing.

¹⁷ For the energy equation, the change in \bar{E}_T is divided by $E_{T,r} = \rho_r \bar{R} T_r / (\gamma_r - 1) + \rho_r u_r^2 / 2$, so that it is the same order of magnitude as the other conservation variables.

In computing the residuals, the summations, maximums, and averages are over all interior grid points, plus points on spatially periodic boundaries.

To avoid undesirably long tables, the convergence parameters are printed at an interval that limits the printout to NHMAX time levels. NHMAX can be specified by the user in namelist IO. However, the residuals are always printed at the first two time levels. This is done because the residuals at time level 1 (the initial condition level) may not be truly representative of the degree of convergence. For instance, if the initial conditions are zero velocity and constant pressure and temperature at every interior point, the computed residuals will be exactly zero. When the time marching procedure begins, however, the flow field will start developing in response to the boundary conditions, and the residuals will reach a maximum in the first few time steps. Note that, in the printout, CHGMAX will be zero until time level $n = \text{ICHECK}$. CHGAVG will only be computed when ICTEST = 2, and will be zero until time level $n = \text{NITAVG}$.

As noted in Section 8.1 of Volume 1, adding artificial viscosity changes the original governing partial differential equations. For cases run with artificial viscosity, therefore, the residuals are printed both with and without the artificial viscosity terms included. This may provide some estimate of the overall error in the solution introduced by the artificial viscosity. Convergence is determined by the residuals with the artificial viscosity terms included.

4.1.7 Additional Output

In addition to the output discussed above, various types of additional printout can be generated by the IDEBUG options, as discussed in Section 3.1.4. Various diagnostics may also appear in the standard output file. These are discussed in greater detail in Section 7.0.

4.2 PLOT FILES

The amount of flow field data generated by a Navier-Stokes code is normally much too large to efficiently comprehend by examining printed output. The computed results are therefore generally examined graphically using various post-processing plotting routines. These plotting routines require as input a file or files, generally called plot files, that are written by the flow solver and contain the coordinates and computed flow field data.

Various types of unformatted plot files may be written by &Proteus, as controlled by the input parameter IPLOT discussed in Section 3.1.4. These files are designed for use by either the CONTOUR or PLOT3D plotting programs.¹⁸

CONTOUR is a three-dimensional plotting program developed at NASA Lewis using internal Lewis-developed graphics routines. It currently can be used only at NASA Lewis on the Amdahl 5860 computer using the VM operating system, or from the Scientific VAX Cluster using the VMS operating system. Originally designed for use with three-dimensional Parabolized Navier-Stokes (PNS) codes, it can be used to generate various types of contour and velocity vector plots in computational planes.

PLOT3D (Walatka, Buning, Pierce, and Elson, 1990) is a sophisticated three-dimensional plotting program specifically designed for displaying results of computational fluid dynamics analyses. It is widely used in government, industry, and universities for interactive visualization of complex flow field data generated by CFD analyses. The computational grid is stored in one file, called an XYZ file, and the computed flow field is stored in another file, called a Q file. There are several options within PLOT3D concerning the format of these files. At NASA Lewis, PLOT3D is available on the Silicon Graphics IRIS workstations and on the Scientific VAX Cluster.

It should be noted that, in Fortran, unformatted files are not generally transportable from computer to computer. If the plot files written by *Proteus* are to be used on some other computer (e.g., a graphics workstation), a separate conversion program will normally be required.

¹⁸ If only the last computed time level is of interest, the restart files may also be used for plotting with PLOT3D. See Section 4.4.

4.2.1 CONTOUR Plot File (IPLOT = 1)

With the IPLOT = 1 option in *Proteus*, a plot file is generated for use by CONTOUR using the following Fortran statements:

```

      write (nplot) title
      write (nplot) machr,rer,lr,ur,pr,tr,rhor,rg,gamr
      write (nplot) level,n1,n2,ism,system
      do 10 il = 1,n1
      write (nplot) ((f(ivar,il,i2),ivar=1,14),i2=1,n2)
10    continue

```

All of the above WRITE statements are executed for each time level written into the file. The plot file thus consists of multiple sets of data, each containing the computed results at a single time level. Note that with this option, the value of the time τ is not written into the file.¹⁹

Unless specified otherwise, all of the parameters written into the CONTOUR plot file are nondimensional, with the appropriate reference condition as the nondimensionalizing factor. The parameters are defined as follows:

TITLE	A descriptive title for the problem.
MACHR	Reference Mach number, $M_r = u_r / \sqrt{\gamma \bar{R} T_r}$. This is a real variable.
RER	Reference Reynolds number, $Re_r = \rho_r u_r L_r / \mu_r$.
LR	Reference length L_r in feet (meters). This is a real variable.
UR	Reference velocity u_r in ft/sec (m/sec).
PR	Reference static pressure $p_r = \rho_r \bar{R} T_r / g_c$ in lb _f /ft ² (N/m ²).
TR	Reference temperature T_r in °R (K).
RHOR	Reference density ρ_r in lb _m /ft ³ (kg/m ³).
RG	Gas constant \bar{R} in ft ² /sec ² -°R (m ² /sec ² -K).
GAMR	Reference ratio of specific heats, $\gamma_r = c_{p,r} / c_{v,r}$.
LEVEL	Time level n .
N1	Number of grid points N_1 in the ξ direction.
N2	Number of grid points N_2 in the η direction.
ISYM	A symmetry parameter used in CONTOUR, set equal to 1.
SYSTEM	A coordinate system parameter used in CONTOUR, set equal to 0.
F(1,,)	Set equal to 0.
F(2,,)	Cartesian x coordinate.
F(3,,)	Cartesian y coordinate or cylindrical r coordinate.
F(4,,)	Set equal to 0.
F(5,,)	Velocity in the ξ direction, V_ξ .
F(6,,)	Velocity in the η direction, V_η .
F(7,,)	Static pressure \bar{p}/p_r .
F(8,,)	Static temperature T .

¹⁹ The IPLOT = -1 option, discussed in the next section, is the better one to use for CONTOUR plot files. The IPLOT = 1 option is included only to be consistent with the various PLOT3D and PLOT2D options.

F(9,,)	Mach number $M = \sqrt{u^2 + v^2 + w^2} / \sqrt{\gamma RT}$
F(10,,)	x-velocity u .
F(11,,)	y or r-velocity v .
F(12,,)	Swirl velocity w .
F(13,,)	Static density ρ .
F(14,,)	Total energy per unit volume E_T .

4.2.2 CONTOUR Plot File (IPLOT = - 1)

The CONTOUR plot file generated using the IPLOT = - 1 option is essentially the same as the one discussed in the previous section. There are just two differences. First, the first two records, containing the title and the reference conditions, are written only once, at the beginning of the file, and not at each time level. And second, the time $\tau_{i,j}$ is written in the F(1,) position. (The subscripts i and j represent grid point indices in the ξ and η directions.)

As noted in the previous section, CONTOUR was originally designed for use with three-dimensional Parabolized Navier-Stokes codes. With PNS codes, the streamwise marching coordinate was written into the F(1,) position, and contours or velocity vectors could be plotted at different streamwise stations.

With the IPLOT = - 1 option in *Proteus*, the resulting CONTOUR plot file is analogous to the one produced for PNS codes, but with the streamwise marching coordinate replaced by the time. Contours and velocity vectors can thus be easily plotted at different time levels.

4.2.3 PLOT3D/WHOLE Plot Files (IPLOT = 2)

With the IPLOT = 2 option in *Proteus*, the XYZ and Q files are written in PLOT3D/WHOLE format. With this option, the XYZ file is written using the following Fortran statements:

```

write (nplotx) n1,n2,n3
write (nplotx) ((x(il,i2),il=1,n1),i2=1,n2),
$              ((y(il,i2),il=1,n1),i2=1,n2),
$              ((zdu      ,il=1,n1),i2=1,n2)

```

The Q file is written using the following Fortran statements:

```

write (nplot) n1,n2,n3
write (nplot) machr,aoa,rer,tau(1,1)
write (nplot) (((qplot(il,i2,ivar),il=1,n1),i2=1,n2),ivar=1,5)

```

The above WRITE statements for the Q file are executed for each time level written into the file. The Q file thus consists of multiple sets of data, each containing the computed results at a single time level. The XYZ file is written only once.²⁰

The parameters written into the file are defined as follows:

N1	Number of grid points N_1 in the ξ direction.
N2	Number of grid points N_2 in the η direction.
N3	In PLOT3D, the number of grid points in the ζ direction. Set equal to 1 for 2-D <i>Proteus</i> .
X	Cartesian x coordinate.
Y	Cartesian y coordinate or cylindrical r coordinate.

²⁰ The current version of PLOT3D does not work for multiple time levels, although future versions might. You can, however, fake it out using the IPLOT = - 3 option described in Section 4.2.5.

ZDUM	In PLOT3D, the Cartesian z coordinate. Set equal to 0. in 2-D <i>Proteus</i> .
MACHR	Reference Mach number, $M_r = u_r / \sqrt{\gamma R T_r}$. This is a real variable.
AOA	In PLOT3D, the angle of attack. Set equal to 0. in <i>Proteus</i> .
RER	Reference Reynolds number, $Re_r = \rho_r u_r L_r / \mu_r$.
TAU(1,1)	The time $\tau_{1,1}$. ²¹
QPLOT(,1)	Static density ρ .
QPLOT(,2)	x-momentum $\rho u M_r$.
QPLOT(,3)	y or r-momentum $\rho v M_r$.
QPLOT(,4)	Swirl momentum $\rho w M_r$.
QPLOT(,5)	Total energy per unit volume $E_T M_r^2$.

All of the parameters written into the XYZ and Q files are nondimensional. However, PLOT3D assumes that velocity is nondimensionalized by the reference speed of sound $a_r = (\gamma R T_r)^{1/2}$, and that energy is nondimensionalized by $\rho_r a_r^2$. In *Proteus* these variables are nondimensionalized by u_r and $\rho_r u_r^2$. That is why M_r appears in the definitions of QPLOT(,2) through QPLOT(,5).

4.2.4 PLOT3D/PLANES Plot Files (IPLOT = 3)

Since *Proteus* 2-D is two-dimensional, the IPLOT = 3 option creates XYZ and Q files identical to those created using the IPLOT = 2 option described in the previous section.

4.2.5 PLOT3D/PLANES Plot Files (IPLOT = - 3)

This option is similar to the IPLOT = 3 option, except that the time τ is written into the z slot in the XYZ file. The XYZ file is written using the following Fortran statements:

```

      write (nplotx) n1,n2,n3
      do 10 i3 = 1,n3
        write (nplotx) (( x(il,i2),il=1,n1),i2=1,n2),
          $              (( y(il,i2),il=1,n1),i2=1,n2),
          $              ((tau(il,i2),il=1,n1),i2=1,n2)
      10 continue

```

The Q file is written using:

```

      write (nplot) n1,n2,n3
      write (nplot) machr,aoa,rer,tdum
      do 20 i3 = 1,n3
        write (nplot) (((qplot(il,i2,ivar),il=1,n1),i2=1,n2),ivar=1,5)
      20 continue

```

The parameters written into the file that have not yet been defined are:

N3	Number of time levels written into the XYZ and Q files. ²²
TAU	The time $\tau_{i,j}$.
TDUM	Set equal to 0.

²¹ Note that with IDTAU = 5 or 6, τ will vary in space, and therefore $\tau_{i,j} \neq \tau_{1,1}$.

²² Note that the number of time levels that end up being written into the files is not known until the end of the *Proteus* run. Therefore, the results are actually written to a scratch file. At the end of the calculation, when N3 is known, the scratch file is read and the XYZ and Q files are written.

Even though a time-dependent version of PLOT3D is not yet available, the IPLOT = - 3 option allows plots to be generated at different time levels by plotting at different PLOT3D "z" stations.

4.2.6 PLOT2D Plot Files (IPLOT = 4)

This option generates XYZ and Q files in PLOT3D's 2D format. The XYZ file is written using the following Fortran statements:

```
write (nplotx) n1,n2
write (nplotx) ((x(il,i2),il=1,n1),i2=1,n2),
$              ((y(il,i2),il=1,n1),i2=1,n2)
```

The Q file is written using:

```
write (nplot) n1,n2
write (nplot) machr,aoa,rer,tau(1,1)
write (nplot) (((qplot(il,i2,ivar),il=1,n1),i2=1,n2),ivar=1,4)
```

As in the IPLOT = 2 option, the above WRITE statements for the Q file are executed for each time level written into the file. The Q file thus consists of multiple sets of data, each containing the computed results at a single time level. The XYZ file is written only once.²⁰

All of the parameters written into the files are the same as previously defined, except:

QPLOT(.,4) Total energy per unit volume $E_T M^2$.

Note that with this option, the swirl momentum ρw is not written into the Q file.

4.3 CONVERGENCE HISTORY FILE

In Section 4.1.6, the convergence history printout is described. The information in this printout is read from an unformatted convergence history file that is updated whenever convergence is checked during a *Proteus* calculation. Plotting routines are also available at NASA Lewis to plot any of the convergence parameters as a function of time level.

The file is written in subroutine RESID using the following Fortran statements. At the first time step of the run,

```
write (nhist) n1,n2,neq,idtau,ictest,nitavg,iswirl,ihstag,
$           iav2e,iav4e,ur,lr,(eps(ieq),ieq=1,neq)
```

Then, at each time level that convergence is being checked,

```
write (nhist) it,tau(1,1),icheck
write (nhist) (chgmax(ieq,1),chgavg(ieq),resl2(ieq,1),
$           resavg(ieq,1),resmax(ieq,1),
$           lrmax(1,ieq,1),lrmax(2,ieq,1),ieq=1,neq)
```

Finally, again at each time level that convergence is being checked, but only for cases run with explicit artificial viscosity,

```
write (nhist) (chgmax(ieq,1),chgavg(ieq),resl2(ieq,2),
$           resavg(ieq,2),resmax(ieq,2),
$           lrmax(1,ieq,2),lrmax(2,ieq,2),ieq=1,neq)
```

The parameters written into the file are defined as follows:

N1	Number of grid points N_1 in the ξ direction.
N2	Number of grid points N_2 in the η direction.
NEQ	The number of coupled governing equations N_{eq} being solved.

IDTAU	Flag for time step selection method.
ICTEST	Flag for type of convergence test.
NITAVG	Number of time steps used in the moving average convergence test.
ISWIRL	Flag for swirl in axisymmetric flow.
IHSTAG	Flag for constant stagnation enthalpy option.
IAV2E	Flag for second-order explicit artificial viscosity.
IAV4E	Flag for fourth-order explicit artificial viscosity.
UR	Reference velocity u_r .
LR	Reference length L_r . This is a real variable.
EPS(IEQ)	Value ϵ used to determine convergence.
IT	Current time level n .
TAU(1,1)	Current value of the time marching parameter $\tau_{1,1}$ at $\xi = \eta = 0$.
ICHECK	Results are checked for convergence every ICHECK'th time level.
CHGMAX(IEQ,1)	Absolute value of the maximum change in the dependent variables from time level $n - 1$ to n .
CHGAVG(IEQ)	Average of the absolute value of the maximum change in the dependent variables for the last NITAVG time steps.
RESL2(IEQ,IAVR)	The L_2 norm of the residual at time level n .
RESAVG(IEQ,IAVR)	The average absolute value of the residual at time level n .
RESMAX(IEQ,IAVR)	The maximum absolute value of the residual at time level n .
LRMAX(IDIR,IEQ,IAVR)	The grid indices (i, j) corresponding to the location of R_{max} .

In the above definitions, the subscript $IEQ = 1$ to N_{eq} , corresponding to the N_{eq} governing equations, $IAVR = 1$ or 2 , corresponding to residuals computed without and with the artificial viscosity terms, and $IDIR = 1$ or 2 , corresponding to the ξ and η directions.

4.4 RESTART FILES

It's often necessary or desirable to run a given case in a series of steps, stopping and restarting between each one. This may be done because of limitations in computer resources, or to change an input parameter. This capability is provided in *Proteus* through a restart option. With this option, the computational mesh and the computed flow field are written as unformatted output files at the end of one run, saved on a magnetic disk or tape, and read in as input files at the beginning of the next run. This process is controlled by the input parameters in namelist RSTRT. (See Section 3.1.3).

The restart files are written and read in subroutine REST. The computational mesh is written using the following Fortran statements:

```

n3 = 1
write (nrkout) n1,n2,n3
write (nrkout) (( x(il,i2),il=1,n1),i2=1,n2),
$              (( y(il,i2),il=1,n1),i2=1,n2),
$              (( tau(il,i2),il=1,n1),i2=1,n2)

```

The computed flow field is written using:

```

n3 = 1
write (nrkout) n1,n2,n3
write (nrkout) machr,aoa,rer,tlevel

```

```

write (nrqout) (((q (il,i2,ivar),il=1,n1),i2=1,n2),ivar=1,5)
if (iturb .gt. 19) write (nrqout) (((qt (il,i2,ivar),il=1,n1),
$                               i2=1,n2),ivar=1,2)
write (nrqout) (((ql(il,i2,ivar),il=1,n1),i2=1,n2),ivar=1,5)
if (iturb .gt. 19) write (nrqout) (((qtl(il,i2,ivar),il=1,n1),
$                               i2=1,n2),ivar=1,2)

```

The parameters written into these files are defined as follows:

N1	Number of grid points N_1 in the ξ direction.
N2	Number of grid points N_2 in the η direction.
X	Cartesian x coordinate.
Y	Cartesian y coordinate.
TAU	Computational time τ .
MACHR	Reference Mach number, $M_r = u/\sqrt{\gamma R T_r}$. This is a real variable.
AOA	Set equal to 0.
RER	Reference Reynolds number, $Re_r = \rho u L_r / \mu_r$.
TLEVEL	The current time level n .
Q(.,1)	Static density ρ at time level n .
Q(.,2)	x -momentum $\rho u M_r$ at time level n .
Q(.,3)	y or r -momentum $\rho v M_r$ at time level n .
Q(.,4)	Swirl momentum $\rho w M_r$ at time level n .
Q(.,5)	Total energy per unit volume $E_T M_r^2$ at time level n .
QL(.,1-5)	Same as Q(.,1-5), except at time level $n-1$.
QT(.,1)	Turbulent kinetic energy k at time level n .
QT(.,2)	Turbulent dissipation rate ε at time level n .
QTL(.,1-2)	Same as QT(.,1-2), except at time level $n-1$.

Note that, except for the QT, QL, and QTL variables, these files have the same format as the XYZ and Q files created using the IPLOT = 2 and 3 options. These restart files can thus also be used as XYZ and Q files for the PLOT3D plotting program. The QT, QL, and QTL variables will not be read by PLOT3D. Note also, however, that the reverse is not true. The XYZ and Q files created using the IPLOT = 2 or 3 option may not be used as restart files, since they do not include the QT, QL, and QTL variables.²³

²³ Actually, if the input parameters IPLT and IPLTA are such that only the final time level is written into the Q file, the XYZ and Q files may be used for an "approximate" restart. In this case, *Proteus* will set QL = Q.

5.0 INITIAL CONDITIONS

Initial conditions are required for the mean flow equations throughout the flow field to start the time marching procedure. Although the best choice for initial conditions will be problem-dependent, some general comments can be made. First, for unsteady flows they should represent a real flow field. A converged steady-state solution from a previous run would be a good choice. Second, to minimize the number of iterations required for steady flows, the initial conditions should be reasonably close to the expected final solution. And third, to minimize problems with starting transients it is important that they represent a physically realistic flow.

If the k - ε turbulence model is being used, then initial conditions are also required for k and ε throughout the flow field. Like the mean flow equations, the best choice for the k - ε initial conditions will be highly problem-dependent, and will have great influence on the convergence and the stability of the computation. It should be noted that $k = 0$ and $\varepsilon = 0$ is merely a trivial solution to the k - ε equations (Nichols, 1990), so the calculation should not be started with $k = \varepsilon = 0$ everywhere in the flow field. The initial profiles for k and ε should be realistic and smooth, and they should at least obey the boundary conditions (i.e., approach zero at solid walls, have zero gradient in the far field, etc.) The initial conditions for the mean flow properties (p , u , v , w , and μ_t) should also be realistic for fully turbulent flows. A good place to start a calculation with the k - ε turbulence model would be a restart run from a converged solution of the same flow field with an algebraic turbulence model.

Initial conditions for the mean flow equations may be supplied by a user-written subroutine, called INIT, by a default version of INIT that specifies uniform flow with constant flow properties everywhere in the flow field, or by restart mesh and flow field files written during a previous run.

Initial conditions for the k - ε equations may be supplied by a user-written subroutine, called KEINIT, by a default version of KEINIT that computes k and ε from the mean flow field and the assumption of local equilibrium (i.e., production equals dissipation), or by restart mesh and flow field files written during a previous k - ε calculation.

5.1 USER-WRITTEN INITIAL CONDITIONS

5.1.1 Mean Flow Equations

As stated above, the best choice for initial conditions will be problem-dependent. For this reason *Proteus* does not include a general-purpose routine for setting up initial conditions. Instead, provision is made for a user-written subroutine, called INIT, that sets up the initial conditions.

The time-marching algorithm used in *Proteus* requires initial conditions for ρ , u , v , w , E_T , p , and T .²⁴ These variables may, of course, be computed from many different combinations of known parameters. To make this process reasonably flexible, the user may choose from several combinations exactly which variables subroutine INIT will supply. This choice is determined by the input parameter ICVARS in namelist FLOW. The following tables list the flow field variables to be supplied by subroutine INIT for the various ICVARS options, along with the Fortran variables into which they should be loaded.²⁵ Remember that the initial conditions must be nondimensionalized by the reference conditions listed in Table 3-2. The default value for ICVARS is 2.

²⁴ Fewer variables may actually be needed, depending on the values of the input parameters IHSTAG and ISWIRL.

²⁵ Note that some input variables, like the Mach number M , are not normally saved as separate Fortran variables. To save storage they are to be loaded into existing Fortran variables in INIT. These Fortran variables will later be loaded with their normal values.

When the energy equation is being solved (IHSTAG = 0), and the flow is two-dimensional, or axisymmetric without swirl (IAXI = 0 or ISWIRL = 0), the allowed values are:

ICVARS	Variables Supplied By INIT	Fortran Variables
1	$\rho, \rho u, \rho v, E_T$	RHO, U, V, ET
2	p, u, v, T	P, U, V, T
3	ρ, u, v, T	RHO, U, V, T
4	p, u, v, ρ	P, U, V, RHO
5	c_p, u, v, T	P, U, V, T
6	p, M, α, T	P, U, V, T

When the energy equation is being solved (IHSTAG = 0), and the flow is axisymmetric with swirl (IAXI = 1 and ISWIRL = 1), the allowed values are:

ICVARS	Variables Supplied By INIT	Fortran Variables
1	$\rho, \rho u, \rho v, \rho w, E_T$	RHO, U, V, W, ET
2	p, u, v, w, T	P, U, V, W, T
3	ρ, u, v, w, T	RHO, U, V, W, T
4	p, u, v, w, ρ	P, U, V, W, RHO
5	c_p, u, v, w, T	P, U, V, W, T
6	$p, M, \alpha, \alpha_w, T$	P, U, V, W, T

When constant stagnation enthalpy is assumed (IHSTAG = 1), and the flow is two-dimensional, or axisymmetric without swirl (IAXI = 0 or ISWIRL = 0), the allowed values are:

ICVARS	Variables Supplied By INIT	Fortran Variables
1	$\rho, \rho u, \rho v$	RHO, U, V
2	p, u, v	P, U, V
3	ρ, u, v	RHO, U, V
5	c_p, u, v	P, U, V
6	p, M, α	P, U, V

When constant stagnation enthalpy is assumed (IHSTAG = 1), and the flow is axisymmetric with swirl (IAXI = 1 and ISWIRL = 1), the allowed values are:

ICVARS	Variables Supplied By INIT	Fortran Variables
1	$\rho, \rho u, \rho v, \rho w$	RHO, U, V, W
2	p, u, v, w	P, U, V, W
3	ρ, u, v, w	RHO, U, V, W
5	c_p, u, v, w	P, U, V, W
6	p, M, α, α_w	P, U, V, W

In the above tables, c_p , α_v , and α_w represent static pressure coefficient, flow angle in degrees in the x-y (or x-r) plane, and flow angle in degrees in the x- θ plane, respectively. These parameters are defined as:

$$c_p = \frac{\bar{p} - p_r}{\rho_r u_r^2 / 2g_c}$$

$$\alpha_v = \tan^{-1} \frac{v}{u}$$

$$\alpha_w = \tan^{-1} \frac{w}{u}$$

The *Proteus* subroutine INITC will use the variables supplied by subroutine INIT to compute ρ , u , v , w , and E_T , using perfect gas relationships if necessary. From these variables, the pressure and temperature will then be recomputed using the equation of state in subroutine EQSTAT, overwriting any values specified by the user in INIT. This ensures a consistent set of initial conditions for the time marching procedure.

Subroutine INIT is called once, immediately after the input has been read, the reference and normalizing conditions have been set, and the geometry and mesh have been defined. The user may do anything he or she desires in the subroutine, such as reading files, reading additional namelist input, making computations, etc. Any of the defined Fortran variables in common blocks may be used.²⁶ (Of course, they should not be changed.) The only requirement is that subroutine INIT return to the calling program (which is INITC) the combination of variables specified by ICVARS, defined at every grid point.

Subroutine INIT is also a convenient place to specify point-by-point boundary condition types and values. It's often easier to do this using Fortran coding rather than entering each value into the namelist input file. See Section 9.2 for a test case with a user-written version of subroutine INIT.

5.1.2 k - ϵ Equations

As with the mean flow equations, the best choice for initial conditions for the k - ϵ equations will be problem-dependent. For this reason, *Proteus* does not have a general-purpose routine for setting up initial conditions for k and ϵ . Instead, provision is made for a user-written subroutine, called KEINIT, that supplies the initial values for k and ϵ .

Assuming that the variables ρ , u , v , w , E_T , p , and T at time level n are available for the turbulent flow field under consideration, the k - ϵ turbulence model in *Proteus* also requires k , ϵ , and μ_t values at time levels n and $n - 1$ before the k - ϵ calculation can be started. These variables may be computed by using many different assumptions and techniques, or simply by fitting the available experimental data.

Subroutine KEINIT is used when a calculation is first started from an initial flow field, or when a restart is made from an algebraic turbulence model calculation. It is called once, immediately after the initial conditions for the mean flow have been computed. Subroutine KEINIT must supply the initial values for the Fortran variables KE, E, MUT (omit MUT if the calculation is a restart from a previous algebraic turbulence model calculation), KEL, EL, and MUTL at every grid point. Usually, KEL, EL, and MUTL can be set equal to KE, E, and MUT, respectively. The default version of KEINIT, described below, is a good place to start when developing a user-written version.

5.2 DEFAULT INITIAL CONDITIONS

5.2.1 Mean Flow Equations

A default version of subroutine INIT is built into *Proteus* that specifies uniform flow with constant flow properties everywhere in the flow field. It uses the ICVARS = 2 option (the default value) and reads initial flow field values of p , u , v , w , and T from namelist IC. The defaults for these parameters are 1.0, 0.0, 0.0, 0.0, and 1.0, respectively, resulting in an initial flow field with $\bar{p} = p$, $u = v = w = 0$, and $\bar{T} = T$. If a value for ICVARS other than 2 is set in the input, a warning message is generated and ICVARS is reset to 2.

5.2.2 k - ϵ Equations

A default version of subroutine KEINIT is built into *Proteus* that calculates the k and ϵ initial profiles using the initial mean flow field values of ρ , u , v , w , and p , along with the assumption of local equilibrium (i.e., production equals dissipation). The turbulent viscosity value μ_t is also needed, and is obtained from the Baldwin-Lomax algebraic model. The assumption of local equilibrium has been found to give good initial values for the k profile near a solid wall. In particular, the location and magnitude of the first peak in the k profile is well predicted. Of course, k and ϵ will depend heavily on the initial velocity and turbulent viscosity, and unrealistic initial profiles for k and ϵ might result from bad initial velocity and turbulent viscosity profiles. The k and ϵ values calculated using this subroutine can be extremely small in regions of

²⁶ See Volume 3 for definitions of all the common block variables.

nearly inviscid, freestream flow. Therefore, it is up to the user to make sure that the freestream values of k and ε are appropriate for the problem under consideration, perhaps by modifying the value of the input parameter CKMIN.

5.3 RESTART INITIAL CONDITIONS

If restart mesh and flow field files were created during a previous run by setting $IREST > 0$ in namelist RSTRT, these files can be used to continue the calculation from the point where the previous run stopped. In this case no subroutine INIT is needed. The restart case is run by linking the existing restart mesh and flow field files to Fortran units NRXIN and NRQIN, respectively, and setting $IREST = 2$ or 3 in the input. New restart files will also be written to units NRXOUT and NRQOUT.

As mentioned above, subroutine KEINIT is used when the k - ε computation is done for the first time ($IREST \leq 1$), or when restarting from a previous computation that used an algebraic turbulence model ($IREST = 3$). However, if the previous calculation used the k - ε turbulence model ($IREST = 2$), then the restart files also contain the information needed to continue the k - ε calculation. In this case, subroutine KEINIT is not needed.

When a restart case is being run, the usual namelist input described in Section 3.1 must still be read in. The following input parameters must have the same values as during the original run: IUNITS, IAXI, IHSTAG, ILAMV, ISWIRL, LR, UR, MACHR, TR, RHOR, MUR, RER, KTR, PRLR, GAMR, RG, HSTAGR, N1, N2, IPACK, and SQ. The remaining input parameters either may be changed during a restart, or do not apply to a restart case. Note, however, that for many of the input parameters, such as those specifying the boundary conditions, changing values during a restart may result in temporary "re-starting" transients or even cause the calculation to blow up.

6.0 RESOURCE REQUIREMENTS

Proteus was developed on the Cray X-MP and Y-MP computers at NASA Lewis Research Center. Changes that may be necessary when porting the code to another system are described in Section 6.1. Sections 6.2 and 6.3 discuss the memory and CPU time required to run the code. The values presented in these sections are for version 2.0 of the *Proteus* code, and were derived from tests run on the NASA Lewis Cray Y-MP in November, 1991. At that time the Cray was running UNICOS Version 6.0 and CFT77 4.0.3. UNICOS and CFT77 are described in the *UNICOS User Commands Reference Manual* (Cray Research, Inc., 1991), and in *CFT77 Compiling System, Volume 1: Fortran Reference Manual* (Cray Research, Inc., 1990), respectively. Section 6.4 describes the size and format of the various input and output files used in the code.

6.1 COMPUTER

Proteus should be transportable to other computer systems with minimal changes. With just three known exceptions, the code is written entirely in ANSI standard Fortran 77. The first exception is the use of namelist input. With namelist input, it's relatively easy to create and/or modify input files, to read the resulting files, and to program default values. Since most Fortran compilers allow namelist input, its use is not considered a serious problem. The second exception is the use of *CALL statements to include COMDECK's, which contain the labeled common blocks, in most of the subprograms. This is a Cray UPDATE feature, and therefore the source code must be processed by UPDATE to create a file that can be compiled.²⁷ UPDATE is described in the *UPDATE Reference Manual* (Cray Research, Inc., 1988). Since using the *CALL statements results in cleaner, more readable code, and since many computer systems have an analogous feature, the *CALL statements were left in the program. The third exception is the use of lowercase alphabetic characters in the Fortran source code. This makes the code easier to read, and is a common extension to Fortran 77.

Several library subroutines are called by *Proteus*. SGEFA and SGESL are Cray versions of LINPACK routines. SASUM and SNRM2 are Cray Basic Linear Algebra Subprograms (BLAS). ISAMAX, ISAMIN, and ISRCHEQ are Cray search routines. TREMAIN is a Cray Fortran library routine. All of these routines are described in detail in Section 4.0 of Volume 3. In addition, SGEFA and SGESL are described in *Volume 3: UNICOS Math and Scientific Library Reference Manual* (Cray Research, Inc., 1989b) and by Dongarra, Moler, Bunch, and Stewart (1979); SASUM, SNRM2, ISAMAX, ISAMIN, and ISRCHEQ are described in *Volume 3: UNICOS Math and Scientific Library Reference Manual* (Cray Research, Inc., 1989b); and TREMAIN is described in *Volume 1: UNICOS Fortran Library Reference Manual* (Cray Research, Inc., 1989a). These or similar routines may be available on other systems. If not, equivalent routines will have to be coded.

6.2 MEMORY

The sizes of the dimensioned arrays in *Proteus*, and hence the amount of memory required to run the program, are set using Fortran parameter statements. These parameters are set in COMDECK PARAMS1. Larger or smaller dimensions may be set for the entire program simply by changing the appropriate parameter, and then recompiling the entire program. The parameters are defined as follows:

N1P	Maximum number of grid points in the ξ direction. The current value is 51.
N2P	Maximum number of grid points in the η direction. The current value is 51.
NMAXP	Maximum of N1P and N2P.
NTOTP	Total storage required for a single two-dimensional array (i.e., $N1P \times N2P$).

²⁷ See the example in Section 8.1.

NEQP	Number of coupled equations allowed. The number of equations to be solved depends on the values of the input parameters IHSTAG and ISWIRL, as shown in Table 3-4 in Section 3.0. In the <i>Proteus</i> code, NEQP is initially set equal to 4. Therefore, to run an axisymmetric case with swirl and with the energy equation, the user must first change the value of NEQP in COMDECK PARAMS1 to 5, then recompile the program. <i>Proteus</i> will still work if NEQP is larger than the value of NEQ shown in Table 3-4, but will use more memory than is actually needed.
NEQPM	Maximum number of coupled equations available. This is the largest permissible value for NEQP. The current value is 5.
NAMAX	Maximum number of time steps allowed in the moving average convergence test (the ICTEST = 2 option). The current value is 10.
NBC	Number of boundary conditions per equation. The current value is 2.
NTP	Maximum number of entries in the table of time-dependent boundary condition values. The current value is 10.
NTSEQP	Maximum number of time step sequences in the time step sequencing option. The current value is 10.

The total amount of memory in computer words required to run *Proteus* 2-D, compiled using CFT77, is listed in the following table for various combinations of the dimensioning parameters N1P, N2P, and NEQP.²⁸ On the Cray X-MP and Y-MP, each word is 64 bits long.

N1P	N2P	MEMORY		
		NEQP = 3	NEQP = 4	NEQP = 5
51	51	527,872	585,216	657,920
101	101	1,071,616	1,295,872	1,581,568
201	201	3,223,040	4,111,872	5,243,392
401	401	11,786,752	15,324,160	19,826,688
51	101	711,168	824,320	968,704
51	201	1,077,760	1,303,040	1,590,272
51	401	1,810,432	2,260,480	2,832,896
101	201	1,793,024	2,239,488	2,808,320
101	401	3,235,840	4,126,720	5,260,800
201	401	6,086,144	7,859,200	10,116,096

6.3 CPU TIME

Compilation of *Proteus* 2-D using the CFT77 compiler requires about 159 seconds of CPU time. The CPU time required for execution will depend on several factors, such as the turbulence model being used, and whether or not the energy equation is being solved. For the test cases in Section 9.0, the CPU time ranged from about 2×10^{-5} to 5×10^{-5} seconds per grid point per time step.

6.4 INPUT/OUTPUT FILES

Several files are used in *Proteus* for various types of input and output. The contents of these files have been described previously in Sections 3.0 and 4.0. This section describes the characteristics of the files themselves. The files are identified by the Fortran variable representing the unit number. The unit numbers have default values, but all of them except NIN may be read in by the user.

Table 6-1 lists the files used in *Proteus*, giving the default unit number, briefly describing the contents of the file, and indicating when it is used. Table 6-2 summarizes the computational resources needed for each file. In this table, the record length is specified in bytes for units NIN and NOUT, and in computer

²⁸ It should be noted that version 4.0.3 of the CFT77 compiler itself requires just under 1,300,000 words.

words for the remaining units. The total file size is specified in printed pages for unit NOUT, and in computer words for the remaining units. Several symbols and Fortran variables are used in Table 6-2, and are defined as:

N_{av}	0 if explicit artificial viscosity is not being used, 1 if it is.
N_{urb}	0 if the input parameter ITURB = 0 or 1, 1 if ITURB = 20.
N_{eq}	The number of coupled equations being solved.
N_t	The number of time levels written into the plot file. This is determined by the input parameters IPLT and IPLTA.
N_{t1}, N_{t2}	The time level at the beginning and at the end of the calculation.
N_η, N_ξ	The number of grid points in the η and ξ directions at which output is being printed. This is determined by the input parameters IPRT1, IPRT2, IPRT1A, and IPRT2A.
N_1, N_2	The number of grid points in the ξ and η directions.
ICHECK	Input parameter specifying frequency for checking convergence.
IPLOT	Input flag specifying type of plot file being written.
NG1, NG2	Number of points in the ξ and η directions in the coordinate system file.

The typical record length and total size values listed in the table were computed assuming $N_{av} = 1$, $N_{urb} = 0$, $N_{eq} = 4$, $N_t = 1$, $N_{t1} = 1$, $N_{t2} = 2000$, $N_\eta = N_\xi = 26$, $N_1 = N_2 = 51$, ICHECK = 10, and NG1 = NG2 = 51.

TABLE 6-1. - I/O FILE TYPES

UNIT	DEFAULT UNIT NO.	RECORD FORMAT	CONTENTS	WHEN USED
NIN	5	Formatted	Standard input	Always
NOUT	6	Formatted	Standard output	Always
NGRID	7	Unformatted	Coordinate system input	NGEOM = 10
NPLOTX	8	Unformatted	PLOT3D XYZ file output	IPLOT = 2, 3, - 3, 4
NPLOT	9	Unformatted	CONTOUR plot file or PLOT3D Q file output	IPLOT \neq 0
NHIST	10	Unformatted	Convergence history output	Always
NRQIN	11	Unformatted	Restart flow field input	IREST = 2, 3
NRQOUT	12	Unformatted	Restart flow field output	IREST = 1, 2, 3
NRXIN	13	Unformatted	Restart computational coordinates input	IREST = 2, 3
NRXOUT	14	Unformatted	Restart computational coordinates output	IREST = 1, 2, 3
NSCR1	20	Unformatted	Scratch	IPLOT = - 3

TABLE 6-2. - I/O FILE SIZES

UNIT	MAXIMUM RECORD LENGTH ^a		TYPICAL MAXIMUM RECORD LENGTH ^a	TOTAL SIZE ^b	TYPICAL TOTAL SIZE ^b	
NIN	80		80	Variable	Variable	
NOUT	132		132	$\approx (2N_{eq} + 3)$ pages + $\{(N_n + 3)[(N_t - 1)/10 + 1]$ $+ 2\}/55$ pages per variable per time level	11 pages + 2 pages per variable per time level	
NGRID	2(NG1)(NG2)		5,202	$2(NG1)(NG2) + 2$	5,204	
NPLOTX	IPLOT			IPLOT		
	2, 3	$3N_1N_2$	7,803	2, 3	$3N_1N_2 + 3$	7,806
	- 3	$3N_1N_2$	7,803	- 3	$3N_1N_2N_t + 3$	7,806
	4	$2N_1N_2$	5,202	4	$2N_1N_2 + 2$	5,204
NPLOT	IPLOT			IPLOT		
	1	$14N_2$	714	1	$(14N_1N_2 + 32)N_t$	36,446
	- 1	$14N_2$	714	- 1	$(14N_1N_2 + 5)N_t$ $+ 27$	36,446
	2, 3, - 3	$5N_1N_2$	13,005	2, 3, - 3	$(5N_1N_2 + 7)N_t$	13,012
	4	$4N_1N_2$	10,404	4	$(4N_1N_2 + 6)N_t$	10,410
NHIST	$7N_{eq}$		28	$(N_{av} + 1)(7N_{eq})(N_{t2} - N_{t1} + 1)$ $/ICHECK + N_{eq} + 12$	11,216	
NRQIN, NRQOUT	$5N_1N_2$		13,005	$10N_1N_2 + 4N_{turb}N_1N_2 + 7$	26,017	
NRXIN, NRXOUT	$3N_1N_2$		7,803	$3N_1N_2 + 3$	7,806	
NSCR1	$6N_1N_2$		15,606	$6N_1N_2N_t$	15,606	

^a In bytes for units NIN and NOUT, and in computer words for the remaining units.

^b In pages for units NIN and NOUT, and in computer words for the remaining units.

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7.0 DIAGNOSTIC MESSAGES

Various diagnostic messages may be printed by *Proteus* as part of its standard output.²⁹ Most of these concern inconsistent or invalid input, although some describe problems encountered during the calculation itself. Two types of messages may appear - errors and warnings. Error messages are preceded by the characters ***** ERROR**, and indicate either a serious problem or an input error that *Proteus* cannot resolve. Warning messages are preceded by the characters ***** WARNING**, and indicate either a potential problem or a non-standard combination of input parameters. Errors cause the calculation to stop, while warnings do not.

The various error and warning messages are listed and explained in the following two subsections. Italic letters, like *value*, are used to indicate variable values. The subprogram in which the message is printed is given in parentheses at the end of the each explanation.

7.1 ERROR MESSAGES

Both itxi and iteta = 0.

A turbulent flow is being computed, with flags set in namelist TURB to bypass the turbulent viscosity computation in both coordinate directions. This makes no sense. (TURBBL)

Both machr and ur specified.

machr = *value*, *ur* = *value*

Either the reference Mach number or velocity may be specified in namelist FLOW, but not both. (INPUT)

Both prlr and ktr specified.

prlr = *value*, *ktr* = *value*

Either the reference laminar Prandtl number or thermal conductivity may be specified in namelist FLOW, but not both. (INPUT)

Both rer and mur specified.

rer = *value*, *mur* = *value*

Either the reference Reynolds number or viscosity may be specified in namelist FLOW, but not both. (INPUT)

Coordinate system file has ng1 and/or ng2 > max allowed.

ng1 = *value*, *ng2* = *value*

A coordinate system file has been read in, using the NGEOM = 10 option, with more grid points than allowed. The maximum allowed values of NG1 and NG2 are the values of the dimensioning parameters N1P and N2P, respectively. (GEOM)

Grid transformation Jacobian changes sign or = 0.

The nonorthogonal grid transformation Jacobian *J* must be either everywhere positive or everywhere negative. This error indicates that the computational mesh contains crossed or coincident grid lines. The error message is followed by a printout of the Cartesian coordinates, the Jacobian, and the metric coefficients. (METS)

²⁹ The diagnostic messages described in this section are generated by the *Proteus* code itself, and appear as part of the standard output. Any computer system error messages due to floating-point errors, etc., are, of course, system-dependent. On UNIX-based systems, system errors will normally appear in the standard error file.

Illegal option for computational coordinates requested.

ngeom = value

An illegal value of NGEOM has been specified in namelist GMTRY. The legal values are 1, 2, and 10, and are described in Section 3.1.5. (GEOM)

Illegal plot file option requested.

iplot = value

An illegal value of IPLOT has been specified in namelist IO. The legal values are 0, ± 1 , 2, ± 3 , and 4, and are described in Section 3.1.4. (PLOT)

Illegal thin-layer option.

ithin(idir) = value

An illegal value of ITHIN(*idir*) has been specified in namelist FLOW. Here *idir* = 1 or 2, corresponding to the ξ and η directions, respectively. The legal values are 0 and 1, and are described in Section 3.1.6. (INPUT)

Illegal time step selection option requested.

idtau = value

An illegal value of IDTAU has been specified in namelist TIME. The legal values are 1 to 9, and are described in Section 3.1.9. (TIMSTP)

Illegal value for iaxi.

iaxi = value

An illegal value of IAXI has been specified in namelist GMTRY. The legal values are 0 and 1, and are described in Section 3.1.5. (INPUT)

Illegal value for icvars.

icvars = value

An illegal value of ICVARS has been specified in namelist FLOW. The legal values are 1 to 6, and are described in Section 3.1.6. (INITC)

Illegal value for ieuler.

ieuler = value

An illegal value of IEULER has been specified in namelist FLOW. The legal values are 0 and 1, and are described in Section 3.1.6. (INPUT)

Illegal value for ihstag.

ihstag = value

An illegal value of IHSTAG has been specified in namelist FLOW. The legal values are 0 and 1, and are described in Section 3.1.6. (INPUT)

Illegal value for ilamv.

ilamv = value

An illegal value of ILAMV has been specified in namelist FLOW. The legal values are 0 and 1, and are described in Section 3.1.6. (FTEMP)

Illegal value for ildamp.

ildamp = value

An illegal value of ILDAMP has been specified in namelist TURB. The legal values are 0 and 1, and are described in Section 3.1.10. (INPUT)

Illegal value for inner.

inner = value

An illegal value of INNER has been specified in namelist TURB. The legal values are 1 and 2, and are described in Section 3.1.10. (INPUT)

Illegal value for irest.
irest = value

An illegal value of IREST has been specified in namelist RSTRT. The legal values are 0, 1, 2, and 3, and are described in Section 3.1.3. (INPUT)

Illegal value for iswirl.
iswirl = value

An illegal value of ISWIRL has been specified in namelist FLOW. The legal values are 0 and 1, and are described in Section 3.1.6. (INPUT)

Illegal value for itxi or iteta.
itxi, iteta = value value

An illegal value of either ITXI or ITETA has been specified in namelist TURB. The legal values are 0 and 1, and are described in Section 3.1.10. (INPUT)

Illegal value for iunits.
iunits = value

An illegal value of IUNITS has been specified in namelist IO. The legal values are 0 and 1, and are described in Section 3.1.4. (INPUT)

Illegal value for lwall1.
lwall1(j,ibound) = value

Illegal value for lwall2.
lwall2(i,ibound) = value

An illegal value of LWALL1 or LWALL2 has been specified in namelist TURB. Here i and j are the indices in the ξ and η directions, and $ibound = 1$ or 2 , corresponding to the $\xi = 0$ or 1 surface, or the $\eta = 0$ or 1 surface. The legal values are 0 and 1, and are described in Section 3.1.10. (INPUT)

Invalid boundary condition type requested.
jbc1(ieq,ibound) or ibc1(j,ieq,ibound) = value

Invalid boundary condition type requested.
jbc2(ieq,ibound) or ibc2(i,ieq,ibound) = value

These messages result from an invalid boundary condition type being specified in namelist BC for the ξ and/or η direction. Here ieq is the boundary condition equation number; $ibound = 1$ or 2 , corresponding to the $\xi = 0$ or 1 surface, or the $\eta = 0$ or 1 surface; and i and j are the indices in the ξ and η directions. The valid boundary conditions are listed in Table 3-6. (BCDENS, BCF, BCGEN, BCPRES, BCQ, BCTEMP, BCUVEL, BCVDIR, BCVVEL, BCWVEL)

Invalid boundary type requested.
kbc1(ibound) = value

Invalid boundary type requested.
kbc2(ibound) = value

These messages result from an invalid boundary type being specified in namelist BC, for the ξ and/or η direction, when the KBC meta flags are used. Here $ibound = 1$ or 2 , corresponding to the $\xi = 0$ or 1 surface, or the $\eta = 0$ or 1 surface. The valid boundary types are listed in Section 3.1.7. (BCSET)

Invalid grid packing location for Roberts formula.
sq(idir,1) = value

An invalid grid packing location, given by the value of SQ(IDIR,1) in namelist NUM, has been specified. Here $idir = 1$ or 2 , corresponding to the ξ and η directions, respectively. The valid values are 0.0, 0.5, and 1.0, and are described in Section 3.1.8. (INPUT)

Invalid grid packing option.

ipack(idir) = value

An invalid grid packing option, given by the value of IPACK(IDIR) in namelist NUM, has been specified. Here *idir* = 1 or 2, corresponding to the ξ and η directions, respectively. The valid values are 0 and 1, and are described in Section 3.1.8. (PAK)

Invalid grid packing parameter for Roberts formula.

sq(idir,2) = value

An invalid grid packing parameter, given by the value of SQ(*idir*,2) in namelist NUM, has been specified. Here *idir* = 1 or 2, corresponding to the ξ and η directions, respectively. The valid values are > 1, and are described in Section 3.1.8. (INPUT)

Invalid time step selection method for time step sequencing option.

idtau = value, ntseq = value

A time step selection option that adjusts $\Delta\tau$ as the solution proceeds has been specified in namelist TIME in conjunction with the time step sequencing option. If the time step sequencing option is being used, IDTAU must be 1, 3, or 5. (INPUT)

Invalid type of artificial viscosity requested.

iav4e, iav2e, iav2i = value value value

An invalid type of artificial viscosity has been specified in namelist NUM. The valid values are 0, 1, and 2 for IAV4E and IAV2E, and 0 and 1 for IAV2I, and are described in Section 3.1.8. (INPUT)

Invalid type of unsteadiness for boundary condition requested.

jtbc1(ieq,ibound) = value

Invalid type of unsteadiness for boundary condition requested.

jtbc2(ieq,ibound) = value

These messages result from an invalid type of unsteadiness being specified in namelist BC for the boundary conditions in the ξ and/or η direction. Here *ieq* is the boundary condition equation number, and *ibound* = 1 or 2, corresponding to the $\xi = 0$ or 1 surface, or the $\eta = 0$ or 1 surface. The valid values for JTBC1 and JTBC2 are 0, 1, and 2, and are described in Section 3.1.7. (TBC)

Mesh size requested > max allowed.

n1 = value, n2 = value

More grid points have been requested in namelist NUM than are allowed. For non-periodic boundary conditions, the maximum allowed values of N1 and N2 are the values of the dimensioning parameters N1P and N2P, respectively. For spatially periodic boundary conditions, the maximum values are N1P - 1 and N2P - 1. (INPUT)

More time step sequences requested than allowed.

ntseq = value

For the time step sequencing option, the number of time step sequences, specified in namelist TIME, cannot exceed the value of the dimensioning parameter NTSEQP. (INPUT)

neqp not large enough.

neqp, ihstag, iswirl = value value value

The value of the dimensioning parameter NEQP, which sets the maximum number of coupled equations that can be solved, is not large enough. The number of equations to be solved depends on the values of the input parameters IHSTAG and ISWIRL. See Section 6.2 and Table 3-4 in Section 3.0. (INPUT)

Non-existent turbulence model requested.

iturb = value

A non-zero value for ITURB has been specified in namelist FLOW that does not correspond to one of the turbulence models currently available in *Proteus*. The only valid non-zero values for ITURB are 1 and 20, corresponding to the algebraic Baldwin-Lomax and the Chien two-equation turbulence models, respectively. (INITC, MAIN)

Non-positive pressure and/or temperature at time level n

I1	I2	P	T
value	value	value	value

During the solution, a non-positive value for pressure and/or temperature has been computed in subroutine EQSTAT. Up to 50 values will be printed. These values, of course, are non-physical and indicate a failure of the solution. Although the calculation will stop, the standard output and plot file will include this time level, if that is consistent with the "IPRT" and "IPLT" parameters in namelist IO. The restart files will not be written. This failure may be caused by bad initial or boundary conditions, or by too large a time step. (INITC, MAIN)

Number of values in unsteady boundary condition table > max allowed.

ntbc = value

For unsteady boundary conditions, the number of values in the tables of GTBC1 and/or GTBC2 vs. NTBCA, specified in namelist TIME, cannot exceed the value of the dimensioning parameter NTP. (INPUT)

Singular block matrix for b. c. at lower boundary, sweep n

Singular block matrix for b. c. at upper boundary, sweep n

When boundary conditions are specified using the JBC and/or IBC input parameters, zero values may appear on the diagonal of the block tridiagonal coefficient matrix. Subroutine FILTER attempts to rearrange the elements of the boundary condition block submatrices to eliminate any of these zero values. These messages indicate it was unable to do so for the boundary and sweep indicated. This means the diagonal submatrix **B** is singular, which in turn means the specified boundary conditions are not independent of one another. (FILTER)

7.2 WARNING MESSAGES

chgmax > 0.15, cfl cut in half.

chgmax > 0.15, dtau cut in half.

With the IDTAU = 2, 4, and 6 options, the time step is adjusted gradually as the solution proceeds based on the absolute value of the maximum change in the dependent variables. One of these messages may occur if the solution changes very rapidly. (The first message applies to the IDTAU = 2 and 6 options, and the second to the IDTAU = 4 option.) Under these conditions the time step is arbitrarily cut in half, and the solution continues. This may stabilize the calculation, but consideration should be given to rerunning the problem with a smaller time step, especially for unsteady flows. (TIMSTP)

Conflicting boundary conditions specified.

`kbc1(ibound) = value, jbc1(ieq,ibound) = value, ibc1(j,ieq,ibound) = value`
`kbc1(ibound) = value` will be used.

Conflicting boundary conditions specified.

`kbc2(ibound) = value, jbc2(ieq,ibound) = value, ibc2(i,ieq,ibound) = value`
`kbc2(ibound) = value` will be used.

These messages indicate that a boundary type was specified using the KBC parameter, and that individual boundary conditions were also specified for that boundary using the JBC and/or IBC parameters, for one of the ξ and/or η boundaries. For a given boundary, the boundary type may be specified using the KBC parameter, or individual boundary conditions may be specified using the JBC and/or IBC parameters. Here *ibound* = 1 or 2, corresponding to the $\xi = 0$ or 1 surface, or the $\eta = 0$ or 1 surface; *ieq* is the boundary condition equation number; and *i* and *j* are the indices in the ξ and η directions. Boundary conditions will be set using the KBC parameter, and the calculation will continue. See the discussion of boundary condition input in Section 3.1.7. (INPUT)

Conflicting boundary conditions specified.

`jbc1(ieq,ibound) = value, ibc1(j,ieq,ibound) = value`
`jbc1(ieq,ibound) = value` will be used.

Conflicting boundary conditions specified.

`jbc2(ieq,ibound) = value, ibc2(i,ieq,ibound) = value`
`jbc2(ieq,ibound) = value` will be used.

These messages indicate that both surface and point-by-point boundary conditions were specified for the same boundary condition equation number for one of the ξ and/or η boundaries. Each boundary condition on each boundary may be specified for the entire surface using the JBC and GBC parameters, or on a point-by-point basis using the IBC and FBC parameters, but not both. Here *ieq* is the boundary condition equation number; *ibound* = 1 or 2, corresponding to the $\xi = 0$ or 1 surface, or the $\eta = 0$ or 1 surface; and *i* and *j* are the indices in the ξ and η directions. A likely cause of this error message is specifying point-by-point boundary conditions without setting the appropriate JBC parameter equal to -1. The boundary condition will be set using the JBC parameter, and the calculation will continue. See the discussion of boundary condition input in Section 3.1.7. (INPUT)

icvars reset to 2 for default init.

The default version of subroutine INIT is set up assuming ICVARS = 2. If another value of ICVARS is read in, it is automatically reset to 2 and the calculation will continue. (INIT)

Illegal convergence testing option requested.

`ictest = value, reset to ictest = 3`

An illegal value of ICTEST has been specified in namelist TIME. ICTEST will be reset to 3, corresponding to convergence based on the L_2 norm of the residual, and the calculation will continue. The legal values are 1 to 5, and are described in Section 3.1.9. (CONV)

Illegal output requested.

`ivout(n) = value`

An illegal value of IVOUT has been specified in namelist IO. It will be ignored and the calculation will continue. The legal values of IVOUT are listed in Table 3-3. (OUTPUT)

Implicit & nonlinear explicit artificial viscosity both requested.

`iav2i, iav2e, iav4e = value value value`

`cavs2i = value value value value value`

Normally, the nonlinear artificial viscosity model, specified by setting IAV2E and IAV4E = 2, is explicit only. This message is printed when implicit artificial viscosity is requested at the same time. *Proteus* will assume that you know what you are doing and the implicit artificial viscosity will be included. (INPUT)

Invalid debug option specified.
idebug(*i*)

An invalid debug option, number *i*, has been specified in namelist IO. The valid IDEBUG options are 1 to 8, and are described in Section 3.1.4. (INPUT)

Non-standard time difference centering requested.

thc = value value
thx = value value value
thy = value value value
thz = value value value
the = value value value

The Beam-Warming type time differencing used in *Proteus* includes three standard implicit schemes - Euler, trapezoidal, and three-point backward. This message indicates that a combination of time differencing parameters θ_1 , θ_2 , and θ_3 has been specified for at least one of the governing equations that does not correspond to any of the three standard schemes. *Proteus* will assume that you know what you are doing and the specified time differencing parameters will be used. (INPUT)

Spatially varying time step requested with time-accurate differencing scheme.

idtau = value
thc = value value
thx = value value value
thy = value value value
thz = value value value
the = value value value

For steady flows, a spatially varying time step may be used to enhance convergence, and first-order time differencing is recommended. Using a second-order time-accurate differencing scheme should not give wrong results, but is wasteful. For unsteady flows, second-order time-accurate differencing should be used, but only a global (i.e., constant in space) time step makes sense. (INPUT)

Time level may fall outside range of input table for unsteady b. c.

itbeg = value, **itend** = value, **ntbca(1)** = value, **ntbca(ntbc)** = value

General unsteady boundary conditions are being used, and the time levels in the input table of GTBC1 and/or GTBC2 vs. NTBKA do not cover the time levels that will occur during the time marching loop. Here **itbeg** and **itend** are the first and last time levels in the time marching loop, and **ntbc** is the value of the input parameter NTBC. If the time level $n < \text{NTBKA}(1)$, the boundary condition value will be set equal to the first value in the table. Similarly, if $n > \text{NTBKA}(\text{NTBC})$, the value will be set equal to the last value in the table. (TBC)

7.3. OTHER MESSAGES

Converged solution at time level *n*

The solution has converged, according to the criteria specified by the parameters ICTEST and EPS, at the indicated time level. The calculation will stop, and the standard output and plot file will include this time level, if that is consistent with the "IPRT" and "IPLT" parameters in namelist IO. The restart files will be written. (MAIN)

Not yet converged.

The solution has not converged, according to the criteria specified by the parameters ICTEST and EPS, within the number of time steps specified by NTIME. The calculation will stop, and the standard output and plot file will include this time level, if that is consistent with the "IPRT" and "IPLT" parameters in namelist IO. The restart files will be written. (MAIN)

Sorry, ran out of cpu time at time level *n*

The calculation was stopped at the indicated time level because the job was almost out of CPU time. The standard output and plot file will include this time level, if that is consistent with the "IPRT" and "IPLT" parameters in namelist IO. The restart files will be written. (MAIN)

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8.0 JOB CONTROL LANGUAGE

At NASA Lewis, *Proteus* is currently being run on the Cray X-MP and Y-MP computers, with UNICOS 6.0 as the operating system. In this section several general examples are presented showing the UNICOS job control language (JCL) that may be used as starting points when setting up specific cases.³⁰ The individual UNICOS commands are described in detail in the *UNICOS User Commands Reference Manual*, (Cray Research, Inc., 1991). These examples are written for the Bourne shell. Some changes may be necessary if the C shell is being used. It is assumed that the user is familiar with the procedures used at their site to submit jobs to the Cray, and to receive and process the output files.

Each example is given with reference line numbers, which are not part of the actual JCL statement, followed by a line-by-line explanation. Note that in UNICOS, the case (upper or lower) of the letters in commands and arguments is significant. In this respect, the examples should be followed exactly.

8.1 COMPILING THE CODE

In this example, the *Proteus* code is compiled on the Cray. It is assumed that the source code is in the proper format for the Cray UPDATE facility, as described in the *UPDATE Reference Manual* (Cray Research, Inc., 1988).

```
1.      # QSUB -lM 2.0mW -lT 300
2.      # QSUB -mb -me
3.      # QSUB -o temp.out
4.      # QSUB -eo
5.      # QSUB -r EXAMPLE
6.      # QSUB -s /bin/sh
7.      set -x
8.      update -i p2d20.s -n p2d20.u -c p2d20 -f
9.      cft77 -b p2d20.o -d pq -o aggress p2d20.f
10.     rm p2d20.f
```

Lines 1 through 6 are actually Network Queueing System (NQS) commands, not UNICOS commands. They must appear first in the runstream, and begin with a # sign. This first line sets the memory and CPU time limits for the job at 2.0 million words and 300 seconds.

Line 2 tells the Cray to send email when the job starts and finishes.

Line 3 tells the Cray to store the standard output in the file *temp.out*.

Line 4 causes any system error messages to be included in the standard output file.

Line 5 gives the name of the job as EXAMPLE.

Line 6 causes the Bourne shell to be used for this job.

Line 7 causes your UNICOS commands to be printed as part of the output runstream.

Line 8 uses the Cray UPDATE facility to create the files, *p2d20.f*, which contains the complete compilable Fortran code for *Proteus*, and *p2d20.u*, which contains the *Proteus* update program library. The update command uses as input the file *p2d20.s*.

Line 9 compiles the program *p2d20.f*, storing the object code in the file *p2d20.o*.

Line 10 removes the named file.

³⁰ See Section 9.0 for specific examples of actual cases.

8.2 RUNNING THE MASTER FILE

The simplest way to run *Proteus* is shown in this example. The existing master file is being used, without making any changes.

```
1.      # QSUB -lM 1.0mW -lT 60
2.      # QSUB -mb -me
3.      # QSUB -o temp.out
4.      # QSUB -eo
5.      # QSUB -r EXAMPLE
6.      # QSUB -s /bin/sh
7.      set -x
8.      ja
9.      touch plotx
10.     touch plotq
11.     touch chist
12.     touch rq2
13.     touch rx2
14.     touch scr1
15.     ln plotx fort.8
16.     ln plotq fort.9
17.     ln chist fort.10
18.     ln rq2 fort.12
19.     ln rx2 fort.14
20.     ln scr1 fort.20
21.     ln coords fort.7
22.     segldr -o p2d20.ex p2d20.o
23.     p2d20.ex << EOD
```

Standard Proteus input file goes here

```
24.     EOD
25.     rm scr1 p2d20.ex
26.     ja -cslt
```

Lines 1 through 6 are actually Network Queueing System (NQS) commands, not UNICOS commands. They must appear first in the runstream, and begin with a # sign. This first line sets the memory and CPU time limits for the job at 1.0 million words and 60 seconds.

Line 2 tells the Cray to send email when the job starts and finishes.

Line 3 tells the Cray to store the standard output in the file *temp.out*.

Line 4 causes any system error messages to be included in the standard output file.

Line 5 gives the name of the job as EXAMPLE.

Line 6 causes the Bourne shell to be used for this job.

Line 7 causes your UNICOS commands to be printed as part of the output runstream.

Line 8 tells the Cray to begin keeping accounting information for later printing.

Lines 9-14 create empty files (assuming they don't already exist) with the file names as shown.

Lines 15-20 link these files with the indicated Fortran unit numbers. The files are thus the PLOT3D XYZ file, the PLOT3D Q file or CONTOUR plot file, the convergence history file, the output restart flow field and mesh files, and the scratch file used with the IPLOT = -3 option. These lines implicitly open the files for input and output. Fortran OPEN statements are not used in *Proteus*. If the *Proteus* input is such that any of these files are unnecessary (see Table 6-1), then the *touch* and *ln* for those files can be eliminated.

Line 21 links an existing computational coordinate system file with Fortran unit 7. If the input parameter NGEOM \neq 10, this line should be eliminated. If a restart case is being run (IREST = 2 or 3), this line should be replaced by the following two lines:

```
ln rq1 fort.11
ln rx1 fort.13
```

The above lines link existing input restart flow field and computational mesh files with Fortran units 11 and 13.

Line 22 creates an executable file, *p2d20.ex*, from the existing object file *p2d20.o*.

Line 23 actually runs the program. The standard *Proteus* input consists of all the records between line 23 and line 24, which contains the marker "EOD". The input could also be obtained from a file on the Cray. In that case, line 23 should be replaced by:

```
p2d20.ex < input
```

where *input* is the name of the file, and line 24 should be eliminated.

Line 25 removes the named files.

Line 26 causes various accounting information to be printed at the end of the output.

8.3 MODIFYING THE MASTER FILE, FULL UPDATE

This example shows how to run with a temporarily modified version of the master file. In this particular case, the existing master file is modified to increase the dimensioning parameters N1P and N2P, thus allowing more mesh points to be used. Since this affects almost every subroutine, the entire program is updated and recompiled.

```
1.  # QSUB -lM 1.0mW -lT 300
2.  # QSUB -mb -me
3.  # QSUB -o temp.out
4.  # QSUB -eo
5.  # QSUB -r EXAMPLE
6.  # QSUB -s /bin/sh
7.  set -x
8.  ja
9.  touch plotx
10. touch plotq
11. touch chist
12. touch rq2
13. touch rx2
14. touch scr1
15. ln plotx fort.8
16. ln plotq fort.9
17. ln chist fort.10
18. ln rq2 fort.12
19. ln rx2 fort.14
20. ln scr1 fort.20
21. ln coords fort.7
22. cat > mods << EOF
    *id temp
    *d params1.20
    parameter (nlp = 81, n2p = 81)
23. EOF
24. update -p p2d20.u -i mods -c temp -f
25. cft77 -d pq -o aggress temp.f
26. segldr -o temp.ex temp.o
27. temp.ex << EOD

    Standard Proteus input file goes here

28. EOD
29. rm scr1 mods temp.f temp.o temp.ex
30. ja -cslt
```

Lines 1 through 21 are the same as in the example in Section 8.2.

Line 22 creates a Cray file called *mods*. The file will consist of all the records between line 22 and line 23, which contains the marker "EOF". Your Cray UPDATE directives and new code should therefore be inserted between lines 22 and 23. The UPDATE directives and new code could also be obtained from a file on the Cray. In that case, lines 22 and 23 should be eliminated.

Line 24 uses the Cray UPDATE facility to create a file called *temp.f*, which contains the complete Fortran code for the modified version of *Proteus*. The update command uses as input the existing *Proteus* program library *p2d20.u*, and the file *mods* containing the UPDATE directives and new code.

Line 25 compiles the modified program *temp.f*, storing the object code in the file *temp.o*.

Line 26 creates an executable file, *temp.ex*, from the object file *temp.o*.

Line 27 actually runs the program. The standard *Proteus* input consists of all the records between line 27 and line 28, which contains the marker "EOD". The input could also be obtained from a file on the Cray. In that case, line 27 should be replaced by:

```
temp.ex < input
```

where *input* is the name of the file, and line 28 should be eliminated.

Line 29 removes the named files.

Line 30 causes various accounting information to be printed at the end of your output.

8.4 MODIFYING THE MASTER FILE. PARTIAL UPDATE

This example shows how to run with temporarily modified versions of just a few routines. In this particular case, the default version of subroutine INIT is replaced by a user-supplied version, and an additional user-supplied geometry option is added to subroutine GEOM. Since these changes affect only INIT and GEOM, only these subroutines are updated and recompiled.

```

1.  # QSUB -lM 1.0mW -lT 60
2.  # QSUB -mb -me
3.  # QSUB -o temp.out
4.  # QSUB -eo
5.  # QSUB -r EXAMPLE
6.  # QSUB -s /bin/sh
7.  set -x
8.  ja
9.  touch plotx
10. touch plotq
11. touch chist
12. touch rq2
13. touch rx2
14. touch scr1
15. ln plotx fort.8
16. ln plotq fort.9
17. ln chist fort.10
18. ln rq2 fort.12
19. ln rx2 fort.14
20. ln scr1 fort.20
21. ln coords fort.7
22. cat > mods << EOF
    *id temp
    *purgedk init
    *deck init

    User-supplied version of INIT goes here.

    *i geom.95

    New user-supplied geometry option goes here

23. EOF
24. update -p p2d20.u -i mods -c temp -q geom init
25. cft77 -d pq -o aggress temp.f
26. segldr -o temp.ex temp.o p2d20.o
27. temp.ex << EOD

    Standard Proteus input file goes here

28. EOD
```

```
29.  rm scr1 mods temp.f temp.o temp.ex
30.  ja -cslt
```

Lines 1 through 21 are the same as in the example in Section 8.2.

Line 22 creates a Cray file called *mods*. The file will consist of all the records between line 22 and line 23, which contains the marker "EOF". Your Cray UPDATE directives and new code should therefore be inserted between lines 22 and 23. The UPDATE directives and new code could also be obtained from a file on the Cray. In that case, lines 22 and 23 should be eliminated.

Line 24 uses the Cray UPDATE facility to create a file called *temp.f*, which contains the Fortran code for the modified versions of subroutines GEOM and INIT. The update command uses as input the existing *Proteus* program library *p2d20.u*, and the file *mods* containing the UPDATE directives and new code.

Line 25 compiles the modified versions of GEOM and INIT, contained in the file *temp.f*, storing the object code in the file *temp.o*.

Line 26 creates an executable file, *temp.ex*, from the object file *temp.o* containing the modified versions of GEOM and INIT, and from the existing object file *p2d20.o*.

Line 27 actually runs the program. The standard *Proteus* input consists of all the records between line 27 and line 28, which contains the marker "EOD". The input could also be obtained from a file on the Cray. In that case, line 27 should be replaced by:

```
temp.ex < input
```

where *input* is the name of the file, and line 28 should be eliminated.

Line 29 removes the named files.

Line 30 causes various accounting information to be printed at the end of your output.

216 INTENTIONALLY BLANK

9.0 TEST CASES

In this section, four test cases are described in detail. The first, developing Couette flow, is a time-accurate calculation of laminar flow generated in a channel by a moving wall. The second case is flow past a circular cylinder. Both inviscid and laminar viscous flow are computed. The third case is transonic turbulent flow in a converging-diverging channel. And the fourth case is turbulent flow over a flat plate, computed using both the Baldwin-Lomax algebraic turbulence model and the Chien two-equation turbulence model. The discussion of each test case includes a description of the problem, listings of the *Proteus* input file and the JCL, a description of the standard *Proteus* output, and figures illustrating the computed results. All the cases were run using version 2.0 of the *Proteus* code on the NASA Lewis Cray Y-MP, running UNICOS 6.0, and Release 4.0.3 of CFT77.

9.1 DEVELOPING COUETTE FLOW

Problem Description

Couette flow is incompressible laminar flow between two infinite parallel walls, one at rest and one moving with velocity u_w . For $\partial p / \partial x = 0$, the steady-state velocity profile is linear, as shown in Figure 9.1. In this test case the time-development of this flow was computed by starting with $u = 0$ everywhere, and suddenly accelerating the top wall to $u = u_w$.

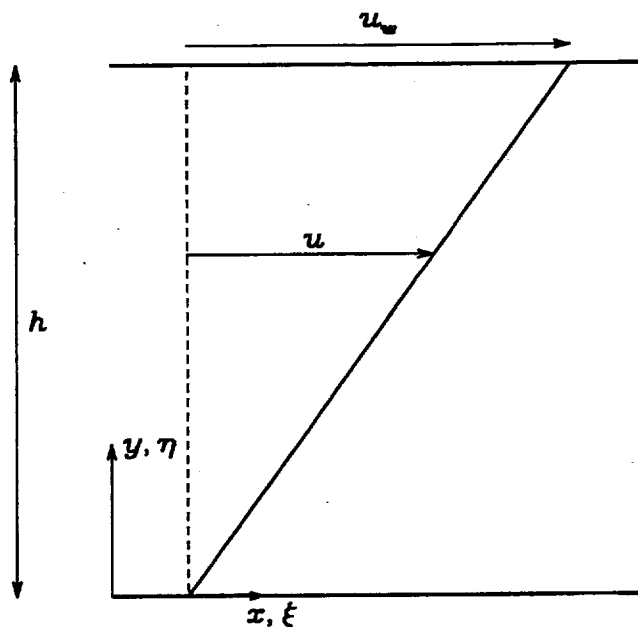


Figure 9.1 - Steady Couette flow with $\partial p / \partial x = 0$.

Reference Conditions

When setting up a problem for *Proteus*, it's usually convenient to first fix the reference conditions. For this case, an obvious choice for the reference length L , was the distance h between the two walls, which

we set equal to 1 ft. Standard sea level conditions of 519 °R and 0.07645 lb_m/ft³ were used for the reference temperature and density. Since *Proteus* is a compressible code, incompressible conditions must be simulated by running at a low Mach number. We therefore set the reference Mach number $M_r = 0.1$. The reference velocity u_r was then computed by *Proteus* from M_r . In setting up the boundary conditions, described below, the velocity at the top wall was set equal to 1. Therefore, $u_r = \bar{u}_w = 0.1\bar{a}$, where \bar{a} is the dimensional speed of sound. In order to reach steady state within a relatively small number of time steps, the reference Reynolds number Re_r was set equal to 100.0.

Computational Coordinates

For this problem a simple Cartesian computational coordinate system can be used. As shown in Figure 9.1, the physical (x - y) and computational (ξ - η) coordinates are thus in the same directions. Since $L_r = \bar{h}$, the coordinate limits in the y direction were $y_{min} = 0$ and $y_{max} = 1$. The solution does not depend on x , so any limits could have been used in the x direction. Convenient values were $x_{min} = 0$ and $x_{max} = 1$.

Initial Conditions

Constant stagnation enthalpy was assumed, so only three initial conditions were required. They were simply $u = v = 0$ and $p = 1$ everywhere in the flow field.

Boundary Conditions

Similarly, only three boundary conditions were required at each computational boundary. Since the solution is independent of x , constant pressure and zero velocity gradient conditions were set at $\xi = 0$ and $\xi = 1$. At both walls, the y -velocity and the normal pressure gradient were set equal to zero. The x -velocity was set equal to zero at the lower wall and 1 at the top wall. These conditions are summarized in the following table.

<u>Boundary</u>	<u>Boundary Conditions</u>
$\xi = 0$	$\partial u / \partial \xi = \partial v / \partial \xi = 0, p = 1$
$\xi = 1$	$\partial u / \partial \xi = \partial v / \partial \xi = 0, p = 1$
$\eta = 0$	$u = v = 0, \partial p / \partial \eta = 0$
$\eta = 1$	$u = 1, v = 0, \partial p / \partial \eta = 0$

Proteus JCL and Input File

The Cray UNICOS job control language used for this case is listed below, including the namelist input. Note that since the defaults for IVOUT(4) and IVOUT(5) are 30 and 40, they are set to zero in namelist IO to avoid printout of the static pressure and temperature. The time levels specified for the printout and plot files correspond to the time values used in a plot of an exact solution to this problem given by Schlichting (1968). Since the solution should not depend on x , only 11 points are used in the ξ direction, and results are printed at only three ξ indices. In namelist FLOW, the only specified reference conditions are MACHR and RER, since the desired values for the remaining ones are the same as the default values. ILAMV is defaulted, resulting in constant viscosity μ and thermal conductivity k . In namelist BC, the JBC values corresponding to derivative boundary conditions are positive, specifying that two-point one-sided differences are to be used. In namelist NUM, the parameters IPACK and SQ are defaulted, which results in an evenly spaced mesh in both directions. The artificial viscosity is turned off because of the low Reynolds number. The second-order three-point backward-implicit time differencing scheme is being used because we want an accurate unsteady solution.³¹ The time step size DT in namelist TIME is simply half of the first time value in the plot of the exact solution mentioned above, and corresponds to a CFL number of about 44.³²

³¹ It should be noted, however, that the incompressible governing equation for this flow is linear. It turns out that even first-order Euler-implicit time differencing gives accurate unsteady results.

³² Smaller time steps were originally used, but it was found that equally good results could be obtained for this case with the input value shown here.

The input includes a brief explanation of each line, and should be compared with the detailed input description in Section 3.0. This JCL is essentially the same as the example presented in Section 8.2, but with lines eliminated that are not applicable to this case.

```
# QSUB -lM 1.0mW -lT 60
# QSUB -mb -me
# QSUB -o temp.out
# QSUB -eo
# QSUB -r cform
# QSUB -s /bin/sh
set -x
ja
touch plotx
touch plotq
touch chist
touch scl
ln plotx fort.8
ln plotq fort.9
ln chist fort.10
ln scl fort.20
segldr -o temp.ex p2d20.o
temp.ex << EOD
Unsteady Developing Couette Flow
&strt
&end
&io
  ivout=1,2,32,47*0,
  iprta=1,3,9,33,73,129,
  iprtla=1,6,11, iprt2=1,
  iplot=-3,
  iplta=3,9,33,73,129,
&end
&gmtry
  ngeom=1,
  xmin=0.0, xmax=1.0,
  ymin=0.0, ymax=1.0,
&end
&flow
  ihstag=1,
  machr=.1, rer=100.,
  gamr=1.4,
&end
&bc
  jbc1(1,1)=41, jbc1(1,2)=41, gbc1(1,1)=1., gbc1(1,2)=1.,
  jbc1(2,1)=12, jbc1(2,2)=12, gbc1(2,1)=0., gbc1(2,2)=0.,
  jbc1(3,1)=22, jbc1(3,2)=22, gbc1(3,1)=0., gbc1(3,2)=0.,
  jbc2(1,1)=42, jbc2(1,2)=42, gbc2(1,1)=0., gbc2(1,2)=0.,
  jbc2(2,1)=11, jbc2(2,2)=11, gbc2(2,1)=0., gbc2(2,2)=1.,
  jbc2(3,1)=21, jbc2(3,2)=21, gbc2(3,1)=0., gbc2(3,2)=0.,
&end
&num
  nl=11, n2=21,
  iav4e=0, iav2e=0, iav2i=0,
  thc=1.0,0.5,
  thx=1.0,0.5,1.0,
  thy=1.0,0.5,1.0,
&end
&time
  idtmod=1000,
  idtau=3, dt=.1953125,
  ntime=1000,
&end
&turb
&end
&ic
```

Not a restart case.

Print u, v, c_p .
Time levels for printout.
Print at 3 ξ indices, all η indices.
Write PLOT3D/PLANES plot files.
Time levels for plot file.

Cartesian computational coordinates.
x-coordinate limits.
y-coordinate limits.

Constant stagnation enthalpy.
Set M , and Re .
Constant specific heats.

$p = 1$ at $\xi = 0, 1$.
 $\partial u / \partial \xi = 0$ at $\xi = 0, 1$.
 $\partial v / \partial \xi = 0$ at $\xi = 0, 1$.
 $\partial p / \partial \eta = 0$ at $\eta = 0, 1$.
 $u = 0, 1$ at $\eta = 0, 1$.
 $v = 0$ at $\eta = 0, 1$.

Use an 11×21 mesh.
No artificial viscosity.

Second-order time differencing.

Don't recompute $\Delta \tau$.
Constant global $\Delta \tau$.
Limit of 1000 time steps.

Laminar flow.

Use default initial conditions.

```

&end
EOD
rm scr1 temp.ex
ja -cslt

```

Standard Proteus Output

The output listing is shown below. In the flow field printout, only the last time level is included. Note that a converged solution is obtained at time level 350, and that this level is automatically included in the standard output and in the plot file.

NASA Lewis Research Center
 Internal Fluid Mechanics Division
 2-d Proteus version 2.0
 January 1992

Unsteady Developing Couette Flow

```

&RSTRT IREST = 0, NRQIN = 11, NRQOUT = 12, NRXIN = 13, NRXOUT = 14, &END
&IO IDEBUG = 20.0, IPLOT = -3, IPLT = 0, IPLTA = 3, 9, 33, 73, 129, 96.0, IPRT = 1, IPRT1 = 1, IPRT2 = 1, IPRTA = 1,
3, 9, 33, 73, 129, 95.0, IPRT1A = 1, 6, 11, 48.0, IPRT2A = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14,
15, 16, 17, 18, 19, 20, 21, 30.0, IUNITS = 0, IVOUT = 1, 2, 32, 47.0, IWOUT1 = 2.0, IWOUT2 = 2.0, NGRID = 7,
NHIST = 10, NHMAX = 100, NOUT = 6, NPLT = 9, NPLTX = 8, NSCR1 = 20, &END
&GMTRY IAXI = 0, NGEOM = 1, RMAX = 1.0, RMIN = 0.0, THMAX = 90.0, THMIN = 0.0, XMAX = 1.0, XMIN = 0.0, YMAX = 1.0,
YMIN = 0.0, &END
&FLOW GAMR = 1.4, HSTAGR = 0.0, ICVARS = 2, IEULER = 0, IHSTAG = 1, ILAMV = 0, ISWIRL = 0, ITHIN = 2.0, KTR = 0.0,
LR = 1.0, MACHR = 0.1, MUR = 0.0, PRLR = 0.0, RER = 100.0, RG = 1716.0, RHOR = 7.645E-02, TR = 519.0, UR = 0.0, &END
&BC FBC1 = 408.0, FBC2 = 408.0, FBCT1 = 204.0, FBCT2 = 204.0, GBC1 = 1.0, 3.0, 1.0, 3.0, GBC2 = 5.0, 1.0,
2.0, GBC1 = 4.0, GBC2 = 4.0, GTBC1 = 80.0, GTBC2 = 80.0, IBC1 = 408.0, IBC2 = 408.0, IBCT1 = 51.0, 51.0,
51.0, 51.0, IBCT2 = 51.0, 51.0, 51.0, 51.0, JBC1 = 41, 12, 22, 0, 41, 12, 22, 0, JBC2 = 42, 11, 21, 0, 42,
11, 21, 0, JBCT1 = 0, 10, 0, 10, JBCT2 = 0, 10, 0, 10, JTBC1 = 8.0, JTBC2 = 8.0, KBC1 = 2.0, KBC2 = 2.0,
NTBC = 0, NTBCA = 10.0, &END
&NUM CAVS2E = 4.1.0, CAVS2I = 4.2.0, CAVS4E = 4.1.0, IAV2E = 0, IAV2I = 0, IAV4E = 0, IPACK = 2.0, N1 = 11, N2 = 21,
SQ = 2.0.0, 2.10000.0, THC = 1.0, 0.5, 0.0, THE = 1.0, 2.0.0, THKE = 1.0, 0.0, THX = 1.0, 0.5, 1.0, THY = 1.0, 0.5,
1.0, THZ = 1.0, 2.0.0, &END
&TIME CFL = 10.1.0, CFLMAX = 10.0, CFLMIN = 0.5, CHG1 = 4.0E-02, CHG2 = 6.0E-02, DT = 0.1953125, 9.1.0E-02, DTF1 = 1.25,
DTF2 = 1.25, DTHMAX = 1.0E-02, DTHMIN = 1.0E-02, EPS = 4.1.0E-03, ICHECK = 10, ICTEST = 3, IDTAU = 3, IDTMOD = 1000,
NDTCYC = 2, NITAVG = 10, NTIME = 1000, 9.0, NTSEQ = 1, TLIN = 20.0, &END
&TURB APLUS = 26.0, CB = 5.5, CCLAU = 1.68E-02, CCP = 1.6, CKLEB = 0.3, CKMIN = 0.0, CMUR = 9.0E-02, CNA = 2.0,
CNL = 1.7, CONE = 1.35, CTWOR = 1.8, CTHREE = 1.15E-02, CVK = 0.4, CWK = 0.25, ILDAMP = 0, INNER = 1, ITETA = 0,
ITURB = 0, ITXI = 1, IWALL1 = 2.0, IWALL2 = 2.0, LWALL1 = -1, 50.0, -1, 50.0, LWALL2 = -1, 50.0, -1, 50.0,
NTKE = 1, PRT = 0.91, REXT1 = 0.0, REXT2 = 0.0, TFACT = 1.0, &END

```

Normalizing conditions

```

Length, lr          = 1.0000E+00 ft
Velocity, ur         = 1.1166E+02 ft/sec
Temperature, tr      = 5.1900E+02 deg R
Density, rhor        = 7.6450E-02 lbm/ft3
Viscosity, mur       = 8.5366E-02 lbm/ft-sec
Thermal conductivity, ktr = 1.0192E-01 lbm-ft/sec3-deg R
Pressure, rhormur**2 = 9.5321E+02 lbm/ft-sec2
Energy/ol., rhormur**2 = 9.5321E+02 lbm/ft-sec2
Gas constant, ur**2/tr = 2.4024E+01 ft2/sec2-deg R
Specific heat, ur**2/tr = 2.4024E+01 ft2/sec2-deg R
Stagnation enthalpy, ur**2 = 1.2468E+04 ft2/sec2
Time, lr/ur         = 8.9556E-03 sec

```

Reference conditions

```

Reynolds number, rer = 1.0000E+02
Mach number, machr   = 1.0000E-01
Specific heat ratio, gamr = 1.4000E+00
Laminar Prandtl number, prlr = 5.0306E+03
"Reference" Prandtl number, prr = 2.0122E+01
Specific heat at const. press. = 6.0060E+03 ft2/sec2-deg R
Specific heat at const. vol. = 4.2900E+03 ft2/sec2-deg R
Gas constant, rg      = 1.7160E+03 ft2/sec2-deg R
Pressure, pr          = 2.1162E+03 lbf/ft2
Stagnation enthalpy, hstagr = 3.1233E+06 ft2/sec2

```

```

&IC P0 = 1.0, T0 = 1.0, U0 = 0.0, V0 = 0.0, W0 = 0.0, &END

```

Boundary condition parameters

```

jbc2(1,2) = 42   gbc2(1,2) = 0.0000E+00
jbc2(2,2) = 11   gbc2(2,2) = 1.0000E+00
jbc2(3,2) = 21   gbc2(3,2) = 0.0000E+00

jbc1(1,1) = 41   gbc1(1,1) = 1.0000E+00   jbc1(1,2) = 41   gbc1(1,2) = 1.0000E+00
jbc1(2,1) = 12   gbc1(2,1) = 0.0000E+00   jbc1(2,2) = 12   gbc1(2,2) = 0.0000E+00
jbc1(3,1) = 22   gbc1(3,1) = 0.0000E+00   jbc1(3,2) = 22   gbc1(3,2) = 0.0000E+00

jbc2(1,1) = 42   gbc2(1,1) = 0.0000E+00
jbc2(2,1) = 11   gbc2(2,1) = 0.0000E+00
jbc2(3,1) = 21   gbc2(3,1) = 0.0000E+00

```

x-velocity at time level 350, time = 6.1045E-01 sec, dtme = 1.7491E-03 sec

```

ixi = 1 6 11
ieta
21 1.0000E+00 1.0000E+00 1.0000E+00
20 9.4988E-01 9.4988E-01 9.4988E-01
19 8.9976E-01 8.9977E-01 8.9977E-01
18 8.4966E-01 8.4966E-01 8.4966E-01
17 7.9956E-01 7.9955E-01 7.9955E-01
16 7.4947E-01 7.4946E-01 7.4946E-01

```

```

15 6.9939E-01 6.9939E-01 6.9938E-01
14 6.4933E-01 6.4932E-01 6.4932E-01
13 5.9928E-01 5.9928E-01 5.9928E-01
12 5.4925E-01 5.4925E-01 5.4925E-01
11 4.9924E-01 4.9924E-01 4.9924E-01
10 4.4925E-01 4.4925E-01 4.4925E-01
9 3.9927E-01 3.9927E-01 3.9927E-01
8 3.4932E-01 3.4932E-01 3.4932E-01
7 2.9938E-01 2.9938E-01 2.9938E-01
6 2.4946E-01 2.4946E-01 2.4946E-01
5 1.9955E-01 1.9955E-01 1.9955E-01
4 1.4965E-01 1.4965E-01 1.4965E-01
3 9.9762E-02 9.9763E-02 9.9765E-02
2 4.9880E-02 4.9880E-02 4.9881E-02
1 0.0000E+00 0.0000E+00 0.0000E+00

y or r-velocity at time level 350, time = 6.1045E-01 sec, dtme = 1.7491E-03 sec

ix1 = 1 6 11
ieta
21 0.0000E+00 0.0000E+00 0.0000E+00
20 -1.7459E-08 1.9716E-07 -1.5651E-07
19 -8.5587E-08 5.3278E-07 -3.3265E-07
18 -1.1763E-07 7.3490E-07 -4.7745E-07
17 -1.3302E-07 6.5559E-07 -5.4990E-07
16 -1.2631E-07 3.0707E-07 -5.7216E-07
15 -1.1410E-07 -1.7391E-07 -5.1893E-07
14 -7.8428E-08 -7.0845E-07 -4.1549E-07
13 -3.2256E-08 -1.2344E-06 -2.7445E-07
12 2.4966E-08 -1.6908E-06 -1.3457E-07
11 7.3243E-08 -2.0388E-06 -1.2009E-08
10 1.1215E-07 -2.2514E-06 7.3234E-08
9 1.3336E-07 -2.3171E-06 1.2360E-07
8 1.4573E-07 -2.2416E-06 1.3875E-07
7 1.4912E-07 -2.0330E-06 1.3171E-07
6 1.5303E-07 -1.7189E-06 1.0759E-07
5 1.5081E-07 -1.3164E-06 7.4553E-08
4 1.4102E-07 -8.7928E-07 3.5631E-08
3 1.0125E-07 -4.4715E-07 -4.8475E-09
2 4.6214E-08 -1.3386E-07 -1.6290E-08
1 0.0000E+00 0.0000E+00 0.0000E+00

Static press. coeff. at time level 350, time = 6.1045E-01 sec, dtme = 1.7491E-03 sec

ix1 = 1 6 11
ieta
21 -2.0258E-02 -1.9536E-02 -2.0259E-02
20 -1.8190E-12 -1.9536E-02 -9.0949E-13
19 -6.3665E-12 -1.9536E-02 -6.3665E-12
18 -1.3642E-11 -1.9536E-02 -1.4552E-11
17 -2.2737E-11 -1.9536E-02 -2.2737E-11
16 -3.2742E-11 -1.9536E-02 -3.1832E-11
15 -4.2746E-11 -1.9536E-02 -4.2746E-11
14 -5.2751E-11 -1.9536E-02 -5.2751E-11
13 -5.9117E-11 -1.9536E-02 -5.9117E-11
12 -6.3665E-11 -1.9536E-02 -6.2755E-11
11 -6.4574E-11 -1.9536E-02 -6.4574E-11
10 -6.2755E-11 -1.9536E-02 -6.3665E-11
9 -6.0027E-11 -1.9536E-02 -5.9117E-11
8 -5.1841E-11 -1.9537E-02 -5.2751E-11
7 -4.2746E-11 -1.9537E-02 -4.3656E-11
6 -3.3651E-11 -1.9537E-02 -3.2742E-11
5 -2.3647E-11 -1.9537E-02 -2.1828E-11
4 -1.3642E-11 -1.9537E-02 -1.3642E-11
3 -6.3665E-12 -1.9537E-02 -7.2760E-12
2 -1.8190E-12 -1.9537E-02 -9.0949E-13
1 -2.0246E-02 -1.9537E-02 -2.0246E-02

**** Converged solution at time level 350
STOP (called by MAIN)

```

This case used 4.6 seconds of CPU time.

Computed Results

As noted earlier, an exact solution exists for this problem (Schlichting, 1968). The solution is in the form of a series of complementary error functions, and is given by³³

$$\begin{aligned}
 \frac{u}{u_w} &= \sum_{n=0}^{\infty} \operatorname{erfc}[(2n+1)\phi_1 - \phi] - \sum_{n=0}^{\infty} \operatorname{erfc}[(2n+1)\phi_1 + \phi] \\
 &= \operatorname{erfc}(\phi_1 - \phi) - \operatorname{erfc}(\phi_1 + \phi) + \operatorname{erfc}(3\phi_1 - \phi) - \operatorname{erfc}(3\phi_1 + \phi) + \dots - \dots
 \end{aligned}$$

where

³³ The solution presented by Schlichting is actually for a stationary top wall and moving bottom wall. The solution presented here, for a stationary bottom wall and moving top wall, was derived from it by replacing y with $h - y$.

$$\phi = \frac{y}{2\sqrt{\nu t}} \sqrt{Re_\tau}$$

$$\phi_1 = \frac{h}{2\sqrt{\nu t}} \sqrt{Re_\tau}$$

The results computed using *Proteus* are compared with the exact solution in Figure 9.2. The results are plotted at times corresponding to $4\sqrt{\nu t} / h\sqrt{Re_\tau} = 0.25, 0.5, 1.0, 1.5,$ and 2.0 , plus the steady state solution.

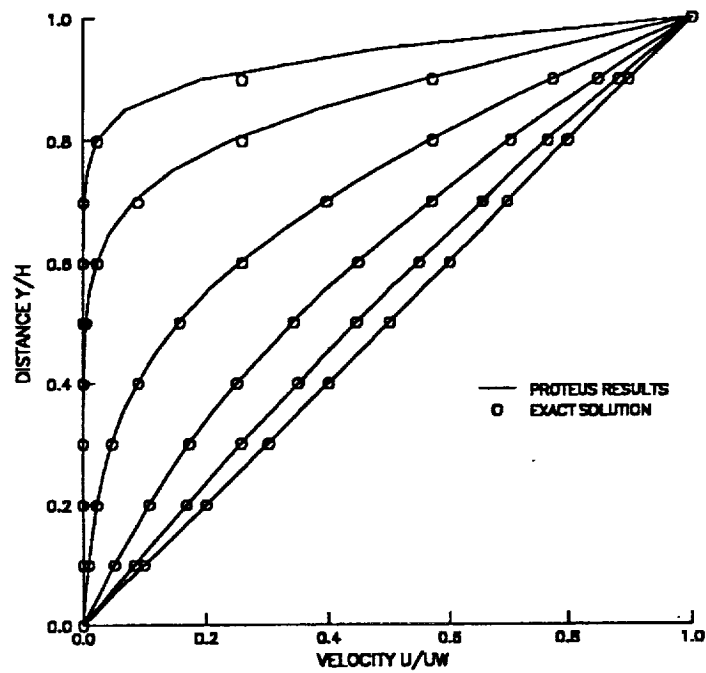


Figure 9.2 - Computed and exact solutions for developing Couette flow.

9.2 FLOW PAST A CIRCULAR CYLINDER

Problem Description

In the second test case, steady flow past a two-dimensional circular cylinder was investigated. Both Euler and viscous flow were computed. The geometric configuration (not to scale) is shown in Figure 9.3.

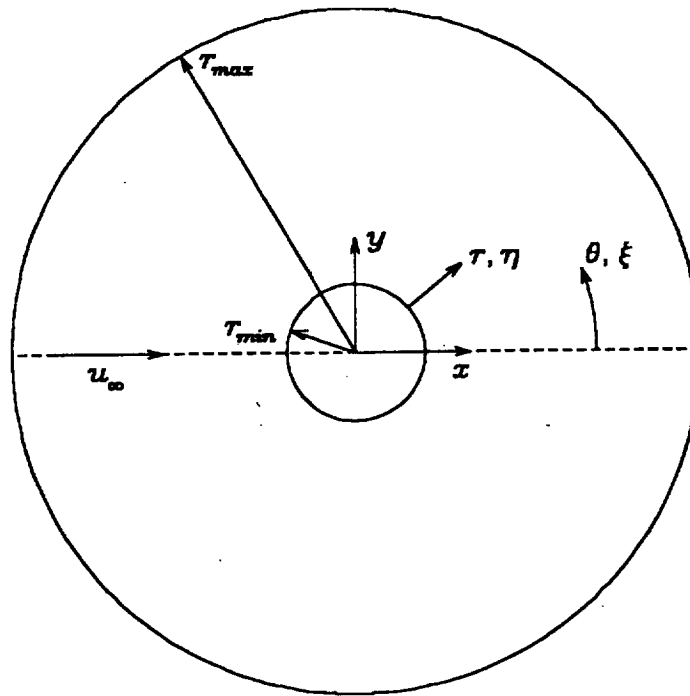


Figure 9.3 - Flow past a circular cylinder.

Reference Conditions

The cylinder radius was used as the reference length L_r , and was set equal to 1 ft. Standard sea level conditions of 519 °R and 0.07645 lb_m/ft³ were used for the reference temperature and density. In order to allow comparison of the *Proteus* results with incompressible experimental data and with potential flow results, the reference Mach number M_r was set to the low value of 0.2. The reference velocity u_r was then computed by *Proteus* from M_r . The experimental data were taken at a Reynolds number based on cylinder diameter of 40. Since our reference length was the cylinder radius, the reference Reynolds number Re_r was 20.

Computational Coordinates

For this problem a polar computational coordinate system was the obvious choice. Figure 9.3 shows the relationship between the physical Cartesian (x - y), physical polar (r - θ), and computational (ξ - η) coordinates. The coordinate limits in the r direction were $r_{min} = 1$ and $r_{max} = 30$. Since the flow is symmetric about the x axis, only the top half of the flow field was computed. The limits in the θ direction were thus $\theta_{min} = 0^\circ$ and $\theta_{max} = 180^\circ$. The $\xi = 0$ and $\xi = 1$ boundaries thus correspond to the $\theta = 0^\circ$ and $\theta = 180^\circ$ boundaries, respectively. The $\eta = 0$ and $\eta = 1$ boundaries correspond to the $r = 1$ and $r = 30$ boundaries.

Initial Conditions

Constant stagnation enthalpy was assumed, so only three initial conditions were required. For the Euler flow case, uniform flow with $u = 1$, $v = 0$, and $p = 1$ was used.

For the viscous flow case, the exact potential flow solution was used to set the initial conditions at all the non-wall points. Thus, with nondimensional free stream conditions of $\rho_\infty = u_\infty = T_\infty = p_\infty = 1$,

$$u = 1 - \frac{1}{r^2} \cos(2\theta)$$

$$v = -\frac{1}{r^2} \sin(2\theta)$$

$$p = (p_T)_\infty - \frac{1}{2} \frac{\rho_\infty (u^2 + v^2)}{R}$$

where

$$(p_T)_\infty = p_\infty + \frac{1}{2} \frac{\rho_\infty u_\infty^2}{R}$$

Note that the nondimensional gas constant R appears in the above equations. This is a result of nondimensionalizing the initial condition for pressure by the reference pressure $p_r = \rho_r \bar{R} T_r$. (See Section 3.1.1). At the cylinder surface, we set the velocities u and v equal to zero, and the pressure p equal to the pressure at the grid point adjacent to the surface. Thus, with two-point one-sided differencing, $\partial p / \partial n = 0$ at the surface.

Boundary Conditions

Again, since we assumed constant stagnation enthalpy, only three boundary conditions were required at each computational boundary. For the Euler flow case, symmetry conditions were used at $\xi = 0$ and $\xi = 1$. At $\eta = 0$, the cylinder surface, the radial velocity and the radial gradient of the circumferential velocity were set equal to zero. The radial gradient of pressure was computed from the polar coordinate form of the incompressible radial momentum equation written at the wall. The equation is (Hughes and Gaylord, 1964)

$$\rho v_r \frac{\partial v_r}{\partial r} + \frac{\rho v_\theta}{r} \frac{\partial v_r}{\partial \theta} - \rho \frac{v_\theta^2}{r} = -\frac{\partial p}{\partial r}$$

where v_r and v_θ are the radial and circumferential velocities, respectively. At the cylinder surface, $v_r = 0$. Thus, at $\eta = 0$,

$$\frac{\partial p}{\partial r} = \rho \frac{v_\theta^2}{r} = \rho \frac{u^2 + v^2}{r}$$

This must be transformed into computational coordinates using

$$\frac{\partial p}{\partial r} = \frac{\partial p}{\partial \eta} \frac{\partial \eta}{\partial r} + \frac{\partial p}{\partial \xi} \frac{\partial \xi}{\partial r}$$

With the computational coordinate system being used for this problem, $\partial \xi / \partial r = 0$. The boundary condition in computational coordinates was thus

$$\frac{\partial p}{\partial \eta} = \frac{\rho v_\theta^2}{r} \left(\frac{\partial \eta}{\partial r} \right)^{-1}$$

where, since $x = r \cos \theta$ and $y = r \sin \theta$,

$$\begin{aligned}\frac{\partial \eta}{\partial r} &= \frac{\partial \eta}{\partial x} \frac{\partial x}{\partial r} + \frac{\partial \eta}{\partial y} \frac{\partial y}{\partial r} \\ &= \frac{\partial \eta}{\partial x} \cos \theta + \frac{\partial \eta}{\partial y} \sin \theta\end{aligned}$$

And finally, at $\eta = 1$ the free stream conditions were specified as boundary conditions. These conditions are summarized in the following table.

Boundary	Boundary Conditions
$\xi = 0$	$\partial u / \partial \xi, v = 0, \partial p / \partial \xi = 0$
$\xi = 1$	$\partial u / \partial \xi, v = 0, \partial p / \partial \xi = 0$
$\eta = 0$	$V_r = 0, \partial V_\theta / \partial \eta = 0, \partial p / \partial \eta = \rho v_\theta^2 / (r \partial \eta / \partial r)$
$\eta = 1$	$u = 1, v = 0, p = 1$

For the viscous flow case, symmetry conditions were again used at $\xi = 0$ and $\xi = 1$. At $\eta = 0$, the cylinder surface, no-slip conditions were used for the velocity, and the radial pressure gradient was set equal to zero. The outer boundary, at $\eta = 1$, was split into an inlet region and wake region. The split was made, somewhat arbitrarily, at $\theta = 45^\circ$. In the inlet region, the boundary values of u , v , and p were kept at their initial values, which were the potential flow values. In the wake region, the boundary values of p were kept at their initial values, and the radial gradients of u and v were set equal to zero. These conditions are summarized in the following table.

Boundary	Boundary Conditions
$\xi = 0$	$\partial u / \partial \xi, v = 0, \partial p / \partial \xi = 0$
$\xi = 1$	$\partial u / \partial \xi, v = 0, \partial p / \partial \xi = 0$
$\eta = 0$	$u = v = 0, \partial p / \partial \eta = 0$
$\eta = 1$ (inlet)	$\Delta u = \Delta v = \Delta p = 0$
$\eta = 1$ (wake)	$\partial u / \partial \eta = \partial v / \partial \eta = 0, \Delta p = 0$

Proteus JCL and Input File for Euler Flow Case

The Cray UNICOS job control language used for the Euler flow case is listed below, including the namelist input. In this test case, we are making a temporary change to the code by supplying a new version of subroutine BCFLIN to implement the boundary condition for pressure at the cylinder surface. BCFLIN is used in conjunction with subroutine BCF for writing boundary conditions that are not among those already built into the code. These routines are described in detail in Volume 3. For the current case, we set $F = p$, $\partial F / \partial \rho = \partial p / \partial \rho$, etc. FBC, the boundary condition value, is set equal to $\rho v_\theta^2 / (r \partial \eta / \partial r)$. In namelist IO, setting IPRT equal to a number larger than the number of time steps to be taken results in a printout at the initial and final time levels only.

The input includes a brief explanation of each line, and should be compared with the detailed input description in Section 3.0. This JCL is essentially the same as the example presented in Section 8.4, but with lines eliminated that are not applicable to this case.

```
# QSUB -lM 1.0mW -lT 60
# QSUB -mb -me
# QSUB -o temp.out
# QSUB -eo
# QSUB -r cylpf
# QSUB -s /bin/sh
set -x
ja
touch plotx
touch plotq
touch chist
ln plotx fort.8
ln plotq fort.9
```

```

In chist fort.10
cat > mods << EOM
*ident mods
*purgedk bcflin
*dk bcflin
      subroutine bcflin (ibc,ieq,ibound,imin,imax,f,dfdrho,dfdru,dfdrv,
      $                  dfdrw,dfdet,fbcm,fbc,fbc)
c
c-----Purpose:  This is a user-supplied subroutine used in conjunction
c                  with the general boundary condition routine bcf (jbc
c                  or ibc options 90-99).  It computes the values needed
c                  for linearization of the boundary condition (i.e., the
c                  values of the function f and its derivatives wrt the
c                  dependent variables).  Note that different user-supplied
c                  boundary conditions can be used at different boundaries
c                  through use of the values of isweep, ieq, and ibound.
c
c                  This version sets the pressure boundary condition at the
c                  surface for inviscid flow over a circular cylinder.  The
c                  pressure gradient in the radial direction is set equal
c                  to the curvature term,  $\rho \times v_{\theta}^2 / r$ , in the radial
c                  momentum equation.  In computational coordinates, the
c                  b. c. is  $dp/d\eta = \rho \times v_{\theta}^2 / (r \times d\eta/dr)$ .
c
c-----Called by:  bcf
c
c-----Calls:
c
c-----Input required:
c
c                  dpdrho, dpdru, - derivatives of pressure with respect to
c                  dpdrv, dpdet,   rho, rho*u, rho*v, et, and rho*w
c                  dpdrw
c
c                  etax, etay - metrics of grid transformation (i.e.,
c                  derivatives of eta wrt x and y)
c
c                  ibc - boundary condition type for current sweep direction
c
c                  ibound - flag specifying boundary; 1 for lower, 2 for upper
c                  (in this application, bcflin will only be called
c                  during the second sweep, and with ibound = 1)
c
c                  ieq - boundary condition equation number (1 to neq)
c
c                  imin, imax - minimum and maximum indices in sweep direction
c
c                  il - current grid point in the xi direction
c
c                  p - static pressure
c
c                  rho, u, v - density, x-velocity, and y-velocity
c
c                  x, y - cartesian coordinates
c
c-----Output computed:
c
c                  dfdrho(iw), dfdru(iw), - derivatives of p with respect to
c                  dfdrv(iw), dfdet(iw),   rho, rho*u, rho*v, et, and rho*w
c                  dfdrw(iw)
c
c                  f(iw) - values of pressure at boundary (iw = 1), at first
c                  point away from boundary (iw = 2), and at second
c                  point away from boundary (iw = 3)
c
c                  fbc - boundary condition values for current sweep direction
c
c-----Common blocks
*call paramsl
*call flowl

```

```

*call metric1
*call num1
      dimension ibc(neqp,nbc),fbc(neqp,nbc)
      dimension f(3),dfdrho(3),dfdru(3),dfdrv(3),dfdrw(3),dfdets(3)
c
c-----Set f, dfdrho, etc., equal to pressure and its derivatives wrt
c-----      dependent variables
c
      do 10 iw = 1,3
        f(iw) = p(il,iw)
        dfdrho(iw) = dpdrho(iw)
        dfdru(iw) = dpdru(iw)
        dfdrv(iw) = dpdrv(iw)
        dfdrw(iw) = dpdrw(iw)
        dfdets(iw) = dpdets(iw)
10      continue
c
c-----Set fbc equal to b. c. value
c
      radius = sqrt(x(il,1)**2 + y(il,1)**2)
      theta = atan2(y(il,1),x(il,1))
      detadr = etax(il,1)*cos(theta) + etay(il,1)*sin(theta)
      vthsq = u(il,1)**2 + v(il,1)**2
      fbc(ieq,ibound) = rho(il,1)*vthsq/radius/detadr
      return
end
EOM
update -p p2d20.u -i mods -c temp -q bcflin
cft77 -d pq -o aggress temp.f
segldr -o temp.ex temp.o p2d20.o
temp.ex << EOD
Euler Flow Past a Circular Cylinder
&rstrt                                     Not a restart case.
&end
&io
      ivout=1,2,32,47*0,
      iprt=10000,
      iprt1=1, iprt2=2,
      iplot=3,
&end
&gmtry
      ngeom=2,
      rmin=1.0, rmax=30.0,
      thmin=0.0, thmax=180.0,
&end
&flow
      ieuler=1,
      ihstag=1,
      machr=.2, rer=20.,
      gamr=1.4,
&end
&bc
      jbc1(1,1)=42, jbc1(1,2)=42, gbc1(1,1)=0., gbc1(1,2)=0.,
      jbc1(2,1)=12, jbc1(2,2)=12, gbc1(2,1)=0., gbc1(2,2)=0.,
      jbc1(3,1)=21, jbc1(3,2)=21, gbc1(3,1)=0., gbc1(3,2)=0.,
      jbc2(1,1)=92, jbc2(1,2)=41, gbc2(1,1)=0., gbc2(1,2)=1.,
      jbc2(2,1)=71, jbc2(2,2)=11, gbc2(2,1)=0., gbc2(2,2)=1.,
      jbc2(3,1)=77, jbc2(3,2)=21, gbc2(3,1)=0., gbc2(3,2)=0.,
&end
&num
      n1=21, n2=51,
      ipack(2)=1,
      sq(2,1)=0., sq(2,2)=1.01,
&end

```

$\partial p / \partial \xi = 0$ at $\xi = 0, 1$.
 $\partial u / \partial \xi = 0$ at $\xi = 0, 1$.
 $v = 0$ at $\xi = 0, 1$.
 $\partial p / \partial \eta = \rho v^2 / (r \partial \eta / \partial r)$.
 $p = 1$ at $\eta = 0, 1$.
 $V_n = 0, u = 1$
at $\eta = 0, 1$.
 $\partial V_i / \partial \eta = 0, v = 0$
at $\eta = 0, 1$.

Use a 21×51 mesh.
 Pack in η direction.
 Pack moderately tightly near $\eta = 0$.

&time	
idtau=5, cfl=10.,	<i>Spatially varying $\Delta\tau$.</i>
ntime=1000,	<i>Limit of 1000 time steps.</i>
&end	
&turb	<i>Laminar flow.</i>
&end	
&ic	
u0=1.,	<i>Uniform flow initial conditions.</i>
&end	
EOD	
rm mods temp.f temp.o temp.ex	
ja -cslt	

Proteus JCL and Input File for Viscous Flow Case

The Cray UNICOS job control language used for the viscous flow case is listed below, including the namelist input. In this case, we are making a temporary change to the code by supplying a new version of subroutine INIT to set the initial conditions described earlier. The procedure for using user-written initial conditions is described in Section 5.1. Note that we are also using subroutine INIT to set point-by-point boundary condition types and values at the outer ($\eta = 1$) boundary, instead of specifying them in namelist BC. It's often easier to set point-by-point boundary conditions in Fortran rather than in the namelist input file.

The input includes a brief explanation of each line, and should be compared with the detailed input description in Section 3.0. This JCL is essentially the same as the example presented in Section 8.4, but with lines eliminated that are not applicable to this case.

```
# QSUB -lM 1.0mW -lT 60
# QSUB -mb -me
# QSUB -o temp.out
# QSUB -eo
# QSUB -r cylv
# QSUB -s /bin/sh
set -x
ja
touch plotx
touch plotq
touch chist
ln plotx fort.8
ln plotq fort.9
ln chist fort.10
cat > mods << EOM
*ident newinit
*purgedk init
*deck init
      subroutine init
c
c-----Purpose:  Set up initial flow field for viscous flow past a
c                  circular cylinder.  The exact potential flow solution
c                  is used as the initial flow field, except at the
c                  cylinder surface where the velocities and normal
c                  pressure gradient are set equal to zero.  This routine
c                  is also used to set point-by-point boundary conditions
c                  at the outer ( $\eta = 1$ ) boundary.
c
c-----Called by:  initc
c
c-----Calls:
c
c-----Input required:
c
c                  icvars - flag specifying which variables are being supplied
c                          as initial conditions
c
c                  neq - number of coupled equations being solved
c
c                  nout - unit number for standard output
```

```

c      n1, n2 - number of grid points in the xi and eta directions
c
c      rgas - gas constant
c
c      x, y - cartesian coordinates
c
c-----Output computed:
c
c      ibc2(il,ieq,2), - point-by-point boundary condition types
c      fbc2(il,ieq,2)   and values at eta = 1 boundary (il = 1 to
c                      nl, ieq = 1 to 3)
c
c      jbc2(ieq,2) - surface boundary condition types at eta = 1
c                  boundary (ieq = 1 to 3)
c
c      p, u, v - initial flow field values of pressure, x-velocity,
c              and y-velocity
c
c      *call params1
c      *call bcl
c      *call flow1
c      *call gmtry1
c      *call iol
c      *call metric1
c      *call num1
c
c-----Check for illegal icvars option (this routine assumes icvars = 2)
c
c      if (icvars .ne. 2) then
c          icvars = 2
c          write (nout,10)
10      format ('' **** warning - icvars reset to 2 in sub. init.'')
c      endif
c
c-----Set freestream conditions
c
c      uinf = 1.
c      rhoinf = 1.
c      tinf = 1.
c      pinf = rhoinf*tinf
c      ptinf = pinf + 0.5*rhoinf*uinf**2/rgas
c      gamma = 1.4
c
c-----Set initial flow field around cylinder equal to exact potential
c-----    flow solution
c
c      do 100 i2 = 2,n2
c      do 100 il = 1,n1
c          radius = sqrt(x(il,i2)**2 + y(il,i2)**2)
c          theta = atan2(y(il,i2),x(il,i2))
c          u(il,i2) = -(cos(2.0*theta)/(radius**2) - 1.0)*uinf
c          v(il,i2) = -(sin(2.0*theta)/(radius**2) )*uinf
c          p(il,i2) = ptinf - 0.5*rhoinf*(u(il,i2)**2 + v(il,i2)**2)/rgas
100      continue
c
c-----Set no-slip and zero gradient conditions at cylinder surface
c
c      do 110 il = 1,n1
c          u(il,1) = 0.0
c          v(il,1) = 0.0
c          p(il,1) = p(il,2)
110      continue
c
c-----Set jbc = -1 for outer boundary to signal use of point-by-point
c-----    boundary conditions
c
c      do 200 ieq = 1, neq
c          jbc2(ieq,2) = -1
200      continue
c

```

```

c-----Set point-by-point boundary conditions at outer boundary
c
      nwake = n1/4
c-----In wake region
      do 210 il = 1,nwake
        ibc2(il,1,2) = 40
        ibc2(il,2,2) = 12
        ibc2(il,3,2) = 22
        fbc2(il,1,2) = 0.
        fbc2(il,2,2) = 0.
        fbc2(il,3,2) = 0.
210    continue
c-----In inlet region
      do 220 il = nwake+1,n1
        ibc2(il,1,2) = 40
        ibc2(il,2,2) = 10
        ibc2(il,3,2) = 20
        fbc2(il,1,2) = 0.
        fbc2(il,2,2) = 0.
        fbc2(il,3,2) = 0.
220    continue
      return
      end

EOM
update -p p2d20.u -i mods -c temp -q init
cft77 -d pq -o aggress temp.f
segldr -o temp.ex temp.o p2d20.o
temp.ex << EOD
VISCOUS FLOW PAST A CIRCULAR CYLINDER
&rstrt                                     Not a restart case.
&end
&io
  ivout=1,2,32,47*0,                      Print u, v, cp.
  iprt=10000,                              Print every 10,000 time levels.
  iprt1=2, iprt2=2,                        Print at every other grid point in both directions.
  iplot=3,                                Write PLOT3D 3-D plot files.
&end
&gmtry
  ngeom=2,                                Polar computational coordinates.
  rmin=1.0, rmax=30.0,                    r-coordinate limits.
  thmin=0.0, thmax=180.0,                 θ-coordinate limits.
&end
&flow
  ihstag=1,                              Constant stagnation enthalpy.
  machr=.2, rer=20.,                     Set Mr and Rer.
  gamr=1.4,                              Constant specific heats.
&end
&bc
  jbc1(1,1)=42, jbc1(1,2)=42, gbc1(1,1)=0., gbc1(1,2)=0.,     $\partial p / \partial \xi = 0$  at  $\xi = 0, 1$ .
  jbc1(2,1)=12, jbc1(2,2)=12, gbc1(2,1)=0., gbc1(2,2)=0.,     $\partial u / \partial \xi = 0$  at  $\xi = 0, 1$ .
  jbc1(3,1)=21, jbc1(3,2)=21, gbc1(3,1)=0., gbc1(3,2)=0.,     $v = 0$  at  $\xi = 0, 1$ .
  jbc2(1,1)=42, gbc2(1,1)=0.,           $\partial p / \partial \eta = 0$  at  $\eta = 0$ .
  jbc2(2,1)=11, gbc2(2,1)=0.,           $u = 0$  at  $\eta = 0$ .
  jbc2(3,1)=21, gbc2(3,1)=0.,           $v = 0$  at  $\eta = 0$ .
&end
&num
  n1=51, n2=51,                          Use a 51 × 51 mesh.
  ipack(2)=1,                             Pack in  $\eta$  direction.
  sq(2,1)=0., sq(2,2)=1.001,             Pack fairly tightly near  $\eta = 0$ .
&end
&time
  idtau=5, cfl=10.,                      Spatially varying  $\Delta \tau$ .
  ntime=1000,                            Limit of 1000 time steps.
&end
&turb
&end

```

EOD

rm mods temp.f temp.o temp.ex
ja -cslt

Standard Proteus Output for Euler Flow Case

The output listing for the Euler flow case is shown below. In the flow field printout, only the last time level is included. Note that a converged solution is obtained at time level 210, and that this level is automatically included in the standard output and in the plot file.

NASA Lewis Research Center
Internal Fluid Mechanics Division

2-d Proteus version 2.0

January 1992

Euler Flow Past a Circular Cylinder

```
&RSTRT IREST = 0, NRQIN = 11, NRQOUT = 12, NRXIN = 13, NRXOUT = 14, &END
&IO IDEBUG = 20*0, IPLOT = 3, IPLT = 0, IPLTA = 101*0, IPRT = 10000, IPRT1 = 1, IPRT2 = 2, IPRTA = 101*0, IPRT1A = 1,
2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 30*0, IPRT2A = 1, 2, 4, 6, 8,
10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48, 50, 51, 24*0, IUNITS = 0,
IWOOUT = 1, 2, 32, 47*0, IWOUT1 = 2*0, IWOUT2 = 2*0, NGRID = 7, NHIST = 10, NHMAX = 100, NOUT = 6, NPLOT = 9,
NPLOTX = 8, NSCR1 = 20, &END
&GMTRY IAXI = 0, NGEOM = 2, RMAX = 30.0, RMIN = 1.0, THMAX = 180.0, THMIN = 0.0, XMAX = 1.0, XMIN = 0.0, YMAX = 1.0,
YMIN = 0.0, &END
&FLOW GAMR = 1.4, HSTAGR = 0.0, ICVARS = 2, IEULER = 1, IMSTAG = 1, ILAMV = 0, ISWIRL = 0, ITHIN = 2*0, KTR = 0.0,
LR = 1.0, MACHR = 0.2, MUR = 0.0, PRLR = 0.0, RER = 20.0, RG = 1716.0, RHOR = 7.645E-02, TR = 519.0, UR = 0.0, &END
&BC FBC1 = 408*0.0, FBC2 = 408*0.0, FBCT1 = 204*0.0, FBCT2 = 204*0.0, GBC1 = 8*0.0, GBC2 = 4*0.0, 2*1.0, 2*0.0,
GBCT1 = 4*0.0, GBCT2 = 4*0.0, GTBC1 = 8*0.0, GTBC2 = 8*0.0, IBC1 = 408*0, IBC2 = 408*0, IBCT1 = 51*0, 51*10, 51*0,
51*10, IBCT2 = 51*0, 51*10, 51*0, 51*10, JBC1 = 42, 12, 21, 0, 42, 12, 21, 0, JBC2 = 92, 71, 77, 0, 41, 11,
21, 0, JBCT1 = 0, 10, 0, 10, JBCT2 = 0, 10, 0, 10, JTBC1 = 8*0, JTBC2 = 8*0, KBC1 = 2*0, KBC2 = 2*0, NTBC = 0,
NTBCA = 10*0, &END
&NUM CAVS2E = 4*1.0, CAVS2I = 4*2.0, CAVS4E = 4*1.0, IAV2E = 0, IAV2I = 1, IAV4E = 1, IPACK = 0, 1, N1 = 21, N2 = 51,
SQ = 2*0.0, 10000.0, 1.01, THC = 1.0, 2*0.0, THE = 1.0, 2*0.0, THKE = 1.0, 0.0, THX = 1.0, 2*0.0, THY = 1.0, 2*0.0,
THZ = 1.0, 2*0.0, &END
&TIME CFL = 10.0, 9*1.0, CFLMAX = 10.0, CFLMIN = 0.5, CHG1 = 4.0E-02, CHG2 = 6.0E-02, DT = 10*1.0E-02, DTF1 = 1.25,
DTF2 = 1.25, DTHAX = 1.0E-02, DTMIN = 1.0E-02, EPS = 4*1.0E-03, ICHECK = 10, ICTEST = 3, IDTAU = 5, IDTMOD = 1,
NDTVC = 2, NITAVG = 10, NTIME = 1000, 9*0, NTSEQ = 1, TLIM = 20.0, &END
&TURB APLUS = 26.0, CB = 5.5, CCLAU = 1.68E-02, CCP = 1.6, CKLEB = 0.3, CKMIN = 0.0, CMUR = 9.0E-02, CNA = 2.0,
CNL = 1.7, CONE = 1.35, CTHOR = 1.0, CTHREE = 1.15E-02, CVK = 0.4, CWK = 0.25, ILDAMP = 0, INNER = 1, ITETA = 0,
ITURB = 0, ITXI = 1, IWALL1 = 2*-1, IWALL2 = 2*-1, LWALL1 = -1, 50*0, -1, 50*0, LWALL2 = -1, 50*0, -1, 50*0,
NTKE = 1, PRT = 0.91, REXT1 = 0.0, REXT2 = 0.0, TFACT = 1.0, &END
```

Normalizing conditions

```
Length, lr = 1.0000E+00 ft
Velocity, ur = 2.2332E+02 ft/sec
Temperature, tr = 5.1900E+02 deg R
Density, rhor = 7.6450E-02 lbm/ft3
Viscosity, mur = 8.5366E-01 lbm/ft-sec
Thermal conductivity, ktr = 1.0192E-01 lbm-ft/sec3-deg R
Pressure, rhormur**2 = 3.8129E+03 lbm/ft-sec2
Energy/vol., rhormur**2 = 3.8129E+03 lbm/ft-sec2
Gas constant, ur**2/tr = 9.6096E+01 ft2/sec2-deg R
Specific heat, ur**2/tr = 9.6096E+01 ft2/sec2-deg R
Stagnation enthalpy, ur**2 = 4.9874E+04 ft2/sec2
Time, lr/ur = 4.4778E-03 sec
```

Reference conditions

```
Reynolds number, rer = 2.0000E+01
Mach number, machr = 2.0000E-01
Specific heat ratio, gamr = 1.4000E+00
Laminar Prandtl number, prlr = 5.0306E+04
"Reference" Prandtl number, prr = 8.0489E+02
Specific heat at const. press. = 6.0060E+03 ft2/sec2-deg R
Specific heat at const. vol. = 4.2900E+03 ft2/sec2-deg R
Gas constant, rg = 1.7160E+03 ft2/sec2-deg R
Pressure, pr = 2.1162E+03 lbf/ft2
Stagnation enthalpy, hstagr = 3.1421E+06 ft2/sec2
```

```
&IC P0 = 1.0, T0 = 1.0, U0 = 1.0, V0 = 0.0, W0 = 0.0, &END
```

Boundary condition parameters

```

jbc2(1,2) = 41 gbc2(1,2) = 1.0000E+00
jbc2(2,2) = 11 gbc2(2,2) = 1.0000E+00
jbc2(3,2) = 21 gbc2(3,2) = 0.0000E+00
jbc1(1,1) = 42 gbc1(1,1) = 0.0000E+00 jbc1(1,2) = 42 gbc1(1,2) = 0.0000E+00
jbc1(2,1) = 12 gbc1(2,1) = 0.0000E+00 jbc1(2,2) = 12 gbc1(2,2) = 0.0000E+00
jbc1(3,1) = 21 gbc1(3,1) = 0.0000E+00 jbc1(3,2) = 21 gbc1(3,2) = 0.0000E+00
jbc2(1,1) = 92 gbc2(1,1) = 0.0000E+00
jbc2(2,1) = 71 gbc2(2,1) = 0.0000E+00
jbc2(3,1) = 77 gbc2(3,1) = 0.0000E+00
```

x-velocity at time level 210

ixi =	1	2	3	4	5	6	7	8	9	10
51	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00
50	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00	9.9977E-01	9.9953E-01	9.9953E-01	9.9973E-01	1.0000E+00	1.0003E+00
48	1.0000E+00	1.0000E+00	1.0000E+00	1.0001E+00	9.9965E-01	9.9983E-01	1.0001E+00	1.0005E+00	1.0009E+00	1.0013E+00
46	1.0000E+00	1.0000E+00	1.0000E+00	1.0002E+00	9.9992E-01	9.9996E-01	1.0003E+00	1.0009E+00	1.0015E+00	1.0020E+00
44	1.0010E+00	1.0010E+00	1.0006E+00	1.0001E+00	9.9980E-01	9.9993E-01	1.0005E+00	1.0013E+00	1.0021E+00	1.0027E+00
42	1.0011E+00	1.0011E+00	1.0006E+00	9.9990E-01	9.9955E-01	9.9982E-01	1.0007E+00	1.0018E+00	1.0028E+00	1.0036E+00

40	1.0010E+00	1.0010E+00	1.0004E+00	9.9952E-01	9.9917E-01	9.9969E-01	1.0009E+00	1.0025E+00	1.0039E+00	1.0048E+00
38	1.0008E+00	1.0008E+00	9.9992E-01	9.9886E-01	9.9953E-01	9.9953E-01	1.0014E+00	1.0035E+00	1.0053E+00	1.0065E+00
36	1.0003E+00	1.0003E+00	9.9904E-01	9.9779E-01	9.9755E-01	9.9933E-01	1.0020E+00	1.0049E+00	1.0074E+00	1.0090E+00
34	9.9905E-01	9.9905E-01	9.9747E-01	9.9608E-01	9.9653E-01	9.9913E-01	1.0030E+00	1.0071E+00	1.0104E+00	1.0126E+00
32	9.9684E-01	9.9684E-01	9.9485E-01	9.9346E-01	9.9479E-01	9.9895E-01	1.0046E+00	1.0102E+00	1.0148E+00	1.0177E+00
30	9.9305E-01	9.9305E-01	9.9063E-01	9.8953E-01	9.9235E-01	9.9882E-01	1.0069E+00	1.0148E+00	1.0210E+00	1.0251E+00
28	9.8689E-01	9.8689E-01	9.8409E-01	9.8379E-01	9.8900E-01	9.9879E-01	1.0103E+00	1.0212E+00	1.0298E+00	1.0355E+00
26	9.7728E-01	9.7728E-01	9.7431E-01	9.7563E-01	9.8448E-01	9.9891E-01	1.0151E+00	1.0301E+00	1.0422E+00	1.0501E+00
24	9.6283E-01	9.6283E-01	9.6013E-01	9.6432E-01	9.7848E-01	9.9925E-01	1.0218E+00	1.0425E+00	1.0592E+00	1.0701E+00
22	9.4184E-01	9.4184E-01	9.4022E-01	9.4911E-01	9.7067E-01	9.9991E-01	1.0309E+00	1.0594E+00	1.0823E+00	1.0972E+00
20	9.1231E-01	9.1231E-01	9.1319E-01	9.2925E-01	9.6071E-01	1.0010E+00	1.0431E+00	1.0817E+00	1.1129E+00	1.1331E+00
18	8.7207E-01	8.7207E-01	8.7784E-01	9.0419E-01	9.4829E-01	1.0025E+00	1.0590E+00	1.1108E+00	1.1526E+00	1.1797E+00
16	8.1892E-01	8.1892E-01	8.3340E-01	8.7372E-01	9.3319E-01	1.0047E+00	1.0792E+00	1.1478E+00	1.2030E+00	1.2388E+00
14	7.5088E-01	7.5088E-01	7.7970E-01	8.3816E-01	9.1545E-01	1.0076E+00	1.1044E+00	1.1935E+00	1.2651E+00	1.3115E+00
12	6.6646E-01	6.6646E-01	7.1689E-01	7.9846E-01	8.9575E-01	1.0113E+00	1.1346E+00	1.2484E+00	1.3396E+00	1.3986E+00
10	5.6505E-01	5.6505E-01	6.4413E-01	7.5565E-01	8.7628E-01	1.0165E+00	1.1693E+00	1.3122E+00	1.4263E+00	1.4999E+00
8	4.4815E-01	4.4815E-01	5.5768E-01	7.0743E-01	8.6031E-01	1.0273E+00	1.2094E+00	1.3834E+00	1.5236E+00	1.6138E+00
6	3.2167E-01	3.2167E-01	4.5159E-01	6.4178E-01	8.4313E-01	1.0497E+00	1.2630E+00	1.4662E+00	1.6316E+00	1.7386E+00
4	1.9759E-01	1.9759E-01	3.2690E-01	5.4037E-01	7.9563E-01	1.0634E+00	1.3259E+00	1.5659E+00	1.7589E+00	1.8827E+00
2	8.8890E-02	8.8890E-02	2.0555E-01	4.1468E-01	6.9636E-01	1.0138E+00	1.3292E+00	1.6149E+00	1.8440E+00	1.9929E+00
1	3.7540E-02	3.9471E-02	1.5509E-01	3.6248E-01	6.5092E-01	9.8023E-01	1.3070E+00	1.6018E+00	1.8378E+00	1.9912E+00

ix1 =	11	12	13	14	15	16	17	18	19	20
51	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00
50	1.0006E+00	1.0007E+00	1.0008E+00	1.0007E+00	1.0006E+00	1.0004E+00	1.0003E+00	1.0001E+00	1.0000E+00	1.0000E+00
48	1.0015E+00	1.0015E+00	1.0014E+00	1.0012E+00	1.0009E+00	1.0006E+00	1.0003E+00	1.0001E+00	9.9999E-01	9.9994E-01
46	1.0022E+00	1.0022E+00	1.0020E+00	1.0017E+00	1.0012E+00	1.0008E+00	1.0004E+00	1.0000E+00	9.9985E-01	9.9979E-01
44	1.0030E+00	1.0030E+00	1.0027E+00	1.0022E+00	1.0016E+00	1.0009E+00	1.0003E+00	9.9989E-01	9.9962E-01	9.9953E-01
42	1.0039E+00	1.0039E+00	1.0035E+00	1.0028E+00	1.0019E+00	1.0010E+00	1.0002E+00	9.9960E-01	9.9922E-01	9.9909E-01
40	1.0052E+00	1.0051E+00	1.0045E+00	1.0036E+00	1.0024E+00	1.0012E+00	1.0000E+00	9.9913E-01	9.9850E-01	9.9838E-01
38	1.0071E+00	1.0069E+00	1.0060E+00	1.0047E+00	1.0031E+00	1.0013E+00	9.9967E-01	9.9838E-01	9.9758E-01	9.9728E-01
36	1.0097E+00	1.0094E+00	1.0082E+00	1.0063E+00	1.0039E+00	1.0014E+00	9.9910E-01	9.9723E-01	9.9605E-01	9.9560E-01
34	1.0134E+00	1.0130E+00	1.0113E+00	1.0086E+00	1.0052E+00	1.0016E+00	9.9821E-01	9.9550E-01	9.9376E-01	9.9308E-01
32	1.0189E+00	1.0182E+00	1.0157E+00	1.0118E+00	1.0069E+00	1.0017E+00	9.9685E-01	9.9292E-01	9.9036E-01	9.8933E-01
30	1.0267E+00	1.0256E+00	1.0219E+00	1.0163E+00	1.0093E+00	1.0018E+00	9.9481E-01	9.8912E-01	9.8538E-01	9.8382E-01
28	1.0376E+00	1.0360E+00	1.0307E+00	1.0226E+00	1.0125E+00	1.0019E+00	9.9183E-01	9.8363E-01	9.7816E-01	9.7583E-01
26	1.0529E+00	1.0504E+00	1.0429E+00	1.0313E+00	1.0170E+00	1.0019E+00	9.8756E-01	9.7580E-01	9.6780E-01	9.6441E-01
24	1.0740E+00	1.0703E+00	1.0596E+00	1.0432E+00	1.0231E+00	1.0018E+00	9.8156E-01	9.6484E-01	9.5347E-01	9.4837E-01
22	1.1023E+00	1.0971E+00	1.0821E+00	1.0593E+00	1.0314E+00	1.0016E+00	9.7332E-01	9.4980E-01	9.3367E-01	9.2626E-01
20	1.1400E+00	1.1327E+00	1.1121E+00	1.0808E+00	1.0425E+00	1.0014E+00	9.6232E-01	9.2962E-01	8.9705E-01	8.8645E-01
18	1.1886E+00	1.1789E+00	1.1511E+00	1.1088E+00	1.0569E+00	1.0013E+00	9.4805E-01	9.0326E-01	8.7215E-01	8.5722E-01
16	1.2508E+00	1.2376E+00	1.2007E+00	1.1445E+00	1.0755E+00	1.0014E+00	9.3016E-01	8.6982E-01	8.2765E-01	8.0694E-01
14	1.3271E+00	1.3099E+00	1.2620E+00	1.1889E+00	1.0989E+00	1.0020E+00	9.0857E-01	8.2876E-01	7.7262E-01	7.4431E-01
12	1.4185E+00	1.3967E+00	1.3357E+00	1.2425E+00	1.1277E+00	1.0037E+00	8.8361E-01	7.8010E-01	7.0676E-01	6.6858E-01
10	1.5247E+00	1.4976E+00	1.4216E+00	1.3055E+00	1.1622E+00	1.0070E+00	8.5605E-01	7.2461E-01	6.3062E-01	5.7977E-01
8	1.6442E+00	1.6114E+00	1.5190E+00	1.3776E+00	1.2027E+00	1.0127E+00	8.2720E-01	6.6393E-01	5.4547E-01	4.7873E-01
6	1.7746E+00	1.7355E+00	1.6257E+00	1.4573E+00	1.2487E+00	1.0214E+00	7.9859E-01	6.0035E-01	4.5422E-01	3.6708E-01
4	1.9233E+00	1.8763E+00	1.7468E+00	1.5491E+00	1.3043E+00	1.0370E+00	7.7412E-01	5.3791E-01	3.5968E-01	2.4717E-01
2	2.0458E+00	1.9977E+00	1.8545E+00	1.6324E+00	1.3551E+00	1.0507E+00	7.4941E-01	4.7590E-01	2.6292E-01	1.2035E-01
1	2.0458E+00	1.9966E+00	1.8496E+00	1.6213E+00	1.3360E+00	1.0224E+00	7.1154E-01	4.2914E-01	2.0626E-01	5.1701E-02

ix1 =	21
51	1.0000E+00
50	1.0000E+00
48	9.9994E-01
46	9.9979E-01
44	9.9953E-01
42	9.9909E-01
40	9.9838E-01
38	9.9728E-01
36	9.9560E-01
34	9.9308E-01
32	9.8933E-01
30	9.8382E-01
28	9.7583E-01
26	9.6441E-01
24	9.4837E-01
22	9.2626E-01
20	8.8645E-01
18	8.5722E-01
16	8.0694E-01
14	7.4431E-01
12	6.6858E-01
10	5.7977E-01
8	4.7873E-01
6	3.6708E-01
4	2.4717E-01
2	1.2035E-01
1	5.7670E-02

y or r-velocity at time level 210

ix1 =	1	2	3	4	5	6	7	8	9	10
51	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
50	0.0000E+00	2.5545E-04	4.6045E-04	4.4817E-04	1.7066E-04	-2.9040E-04	-7.8410E-04	-1.1722E-03	-1.3798E-03	-1.3993E-03
48	0.0000E+00	2.0835E-04	3.4986E-04	2.5821E-04	-8.6962E-05	-5.6570E-04	-1.0004E-03	-1.2517E-03	-1.2708E-03	-1.0921E-03
46	0.0000E+00	1.1258E-04	1.3464E-04	-1.2082E-04	-6.4949E-04	-1.2739E-03	-1.7625E-03	-1.9543E-03	-1.8158E-03	-1.4199E-03
44	0.0000E+00	4.8456E-05	-2.9109E-05	-6.6240E-04	-1.2164E-03	-2.0319E-03	-2.6026E-03	-2.7412E-03	-2.4367E-03	-1.8087E-03
42	0.0000E+00	-3.8884E-05	-2.5956E-04	-9.4569E-04	-2.0068E-03	-3.0636E-03	-3.7131E-03	-3.7467E-03	-2.3779E-03	-2.2538E-03
40	0.0000E+00	-2.0079E-04	-6.6072E-04	-1.7223E-03	-3.2020E-03	-4.5529E-03	-5.2583E-03	-5.1072E-03	-4.2878E-03	-2.8358E-03
38	0.0000E+00	-4.7849E-04	-1.3231E-03	-2.9336E-03	-4.9749E-03	-6.6719E-03	-7.3833E-03	-6.9294E-03	-5.5337E-03	-3.5820E-03
36	0.0000E+00	-9.3744E-04	-2.3856E-03	-4.7877E-03	-7.5721E-03	-9.6628E-03	-1.0298E-02	-9.3835E-03	-7.3028E-03	-4.5670E-03
34	0.0000E+00	-1.6809E-03	-4.0602E-03	-7.5895E-03	-1.1340E-02	-1.3860E-02	-1.4296E-02	-1.2715E-02	-9.7034E-03	-5.9004E-03
32	0.0000E+00	-2.8566E-03	-6.6424E-03	-1.1751E-02	-1.6739E-02	-1.9707E-02	-1.9781E-02	-1.7276E-02	-1.3811E-02	-7.7408E-03
30	0.0000E+00	-4.6725E-03	-1.0536E-02	-1.7820E-02	-2.4366E-02	-2.7789E-02	-2.7305E-02	-2.3565E-02	-1.7615E-02	-1.0300E-02
28	0.0000E+00	-7.4163E-03	-1.6283E-02	-2.6507E-02	-3.4991E-02	-3.8876E-02	-3.7620E-02	-3.2264E-02	-2.4033E-02	-1.3848E-02
26	0.0000E+00	-1.1477E-02	-2.4590E-02	-3.8717E-02	-4.9587E-02	-5.3967E-02	-5.1726E-02	-4.4276E-02	-2.7812E-02	-1.8712E-02
24	0.0000E+00	-1.7373E-02	-3.6357E-02	-5.5561E-02	-6.9345E-02	-7.4326E-02	-7.0900E-02	-6.0724E-02	-4.5106E-02	-2.5288E-02
22	0.0000E+00	-2.5800E-02	-5.2696E-02	-7.8334E-02	-9.5664E-02	-1.0148E-01	-9.6688E-02	-8.2934E-02	-6.1484E-02	-3.3998E-02
20	0.0000E+00	-3.7682E-02	-7.4913E-02	-1.0845E-01	-1.3008E-01	-1.3719E-01	-1.3083E-01	-1.1235E-01	-8.3865E-02	-4.5371E-02
18	0.0000E+00	-5.4288E-02	-1.0444E-01	-1.4728E-01	-1.7417E-01	-1.8328E-01	-1.7510E-01	-1.5040E-01	-1.1866E-01	-5.9938E-02
16	0.0000E+00	-7.6713E-02	-1.4266E-01	-1.9602E-01	-2.2932E-01	-2.4145E-01	-2.3107E-01	-1.9834E-01	-1.4579E-01	-7.8207E-02
14	0.0000E+00	-1.0622E-01	-1.9072E-01	-2.5559E-01	-2.9656E-01	-3.1293E-01	-2.9985E-01	-2.5706E-01	-1.8852E-01	-1.0057E-01


```

12 0.0000E+00 -1.4240E-01 -2.4910E-01 -3.2681E-01 -3.7642E-01 -3.9804E-01 -3.8183E-01 -3.2692E-01 -2.3931E-01 -1.2716E-01
10 0.0000E+00 -1.8194E-01 -3.1616E-01 -4.1048E-01 -4.6953E-01 -4.9614E-01 -4.7630E-01 -4.0761E-01 -2.9792E-01 -1.5779E-01
8 0.0000E+00 -2.1700E-01 -3.8431E-01 -5.0493E-01 -5.7760E-01 -6.0737E-01 -5.8187E-01 -4.9779E-01 -3.6346E-01 -1.9199E-01
6 0.0000E+00 -2.3683E-01 -4.3629E-01 -5.9723E-01 -6.9793E-01 -7.3399E-01 -6.9963E-01 -5.9682E-01 -4.3489E-01 -2.2915E-01
4 0.0000E+00 -2.3826E-01 -4.5500E-01 -6.5810E-01 -8.0380E-01 -8.6248E-01 -8.2577E-01 -7.0418E-01 -5.1217E-01 -2.6888E-01
2 0.0000E+00 -2.4139E-01 -4.6092E-01 -6.8482E-01 -8.6290E-01 -9.4669E-01 -9.1890E-01 -7.9035E-01 -5.7803E-01 -3.0442E-01
1 0.0000E+00 -2.4921E-01 -4.7732E-01 -7.1141E-01 -8.9592E-01 -9.8023E-01 -9.4957E-01 -8.1615E-01 -5.9714E-01 -3.1537E-01

```

```

ixi =      11      12      13      14      15      16      17      18      19      20
ieta
51 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00
50 -1.2661E-03 -1.0323E-03 -7.5001E-04 -4.6644E-04 -2.2092E-04 -4.1604E-05 5.9240E-05 8.8283E-05 6.8766E-05 3.0944E-05
48 -7.9251E-04 -4.5113E-04 -1.2793E-04 1.3826E-04 3.2394E-04 4.1727E-04 4.1936E-04 3.4731E-04 2.3221E-04 1.0861E-04
46 -8.8731E-04 -3.3358E-04 1.5646E-04 5.3127E-04 7.6504E-04 8.5143E-04 8.0229E-04 6.4830E-04 4.3474E-04 2.0823E-04
44 -1.0238E-03 -2.3156E-04 4.6016E-04 9.8382E-04 1.3031E-03 1.4075E-03 1.3123E-03 1.0601E-03 7.1478E-04 3.4486E-04
42 -1.1380E-03 -3.8350E-05 9.1280E-04 1.6292E-03 2.0601E-03 2.1872E-03 2.0280E-03 1.6390E-03 1.1089E-03 5.3689E-04
40 -1.2802E-03 2.3150E-04 1.5377E-03 2.5248E-03 3.1179E-03 3.2841E-03 3.0410E-03 2.4623E-03 1.6708E-03 8.1037E-04
38 -1.4328E-03 6.4317E-04 2.4446E-03 3.8146E-03 4.6392E-03 4.8622E-03 4.5003E-03 3.6508E-03 2.4832E-03 1.2052E-03
36 -1.6042E-03 1.2629E-03 3.7706E-03 5.6904E-03 6.8473E-03 7.1512E-03 6.6185E-03 5.3790E-03 3.6665E-03 1.7799E-03
34 -1.8030E-03 2.1923E-03 5.7169E-03 8.4281E-03 1.0060E-02 1.0477E-02 9.6971E-03 7.8956E-03 5.3937E-03 2.6185E-03
32 -2.0368E-03 3.5803E-03 8.5690E-03 1.2413E-02 1.4721E-02 1.5294E-02 1.4157E-02 1.1549E-02 7.9084E-03 3.8398E-03
30 -2.3042E-03 5.6407E-03 1.2721E-02 1.8172E-02 2.1431E-02 2.2220E-02 2.0573E-02 1.6818E-02 1.1547E-02 5.6077E-03
28 -2.5878E-03 8.6653E-03 1.8698E-02 2.6404E-02 3.0994E-02 3.2084E-02 2.9719E-02 2.4345E-02 1.6746E-02 8.1442E-03
26 -2.8521E-03 1.3032E-02 2.7172E-02 3.8010E-02 4.4451E-02 4.5963E-02 4.2600E-02 3.4970E-02 2.1544E-02 1.1741E-02
24 -3.0506E-03 1.9201E-02 3.8973E-02 5.4112E-02 6.3104E-02 6.5208E-02 6.0483E-02 4.9751E-02 3.4472E-02 1.6768E-02
22 -3.1586E-03 2.7704E-02 5.5079E-02 7.6039E-02 8.8506E-02 9.1441E-02 8.4891E-02 6.9966E-02 4.8634E-02 2.3676E-02
20 -3.0881E-03 3.9117E-02 7.6571E-02 1.0528E-01 1.2240E-01 1.2648E-01 1.1755E-01 9.7074E-02 6.7699E-02 3.2990E-02
18 -2.8930E-03 5.4016E-02 1.0455E-01 1.4335E-01 1.6657E-01 1.7223E-01 1.6029E-01 1.3263E-01 9.2819E-02 4.5290E-02
16 -2.5653E-03 7.2907E-02 1.3999E-01 1.9160E-01 2.2264E-01 2.3045E-01 2.1484E-01 1.7817E-01 1.2516E-01 6.1182E-02
14 -2.1275E-03 9.6133E-02 1.8356E-01 2.5100E-01 2.9181E-01 3.0246E-01 2.8258E-01 2.3497E-01 1.6580E-01 8.1264E-02
12 -1.6106E-03 1.2378E-01 2.3548E-01 3.2187E-01 3.7453E-01 3.8889E-01 3.6430E-01 3.0391E-01 2.1563E-01 1.0610E-01
10 -1.0210E-03 1.5562E-01 2.9534E-01 4.0375E-01 4.7037E-01 4.8943E-01 4.5997E-01 3.8527E-01 2.7529E-01 1.3621E-01
8 -7.8979E-04 1.9113E-01 3.6219E-01 4.9539E-01 5.7796E-01 6.0281E-01 5.6870E-01 4.7867E-01 3.4515E-01 1.7204E-01
6 -1.6329E-04 2.2926E-01 4.3422E-01 5.9443E-01 6.9472E-01 7.2657E-01 6.8847E-01 5.8293E-01 4.2526E-01 2.1390E-01
4 -1.3982E-03 2.7098E-01 5.1230E-01 7.0153E-01 8.2111E-01 8.6106E-01 8.1974E-01 6.9876E-01 5.1686E-01 2.6227E-01
2 2.0312E-03 3.0932E-01 5.8587E-01 8.0427E-01 9.4423E-01 9.9404E-01 9.5184E-01 8.1842E-01 6.1641E-01 3.1555E-01
1 -2.7310E-03 3.1623E-01 6.0098E-01 8.2610E-01 9.7063E-01 1.0224E+00 9.7935E-01 8.4224E-01 6.3481E-01 3.2643E-01

```

```

ixi =      21
ieta
51 0.0000E+00
50 0.0000E+00
48 0.0000E+00
46 0.0000E+00
44 0.0000E+00
42 0.0000E+00
40 0.0000E+00
38 0.0000E+00
36 0.0000E+00
34 0.0000E+00
32 0.0000E+00
30 0.0000E+00
28 0.0000E+00
26 0.0000E+00
24 0.0000E+00
22 0.0000E+00
20 0.0000E+00
18 0.0000E+00
16 0.0000E+00
14 0.0000E+00
12 0.0000E+00
10 0.0000E+00
8 0.0000E+00
6 0.0000E+00
4 0.0000E+00
2 0.0000E+00
1 0.0000E+00

```

Static press. coeff. at time level 210

```

ixi =      1      2      3      4      5      6      7      8      9      10
ieta
51 3.8829E-03 2.2737E-13 2.2737E-13 2.2737E-13 2.2737E-13 2.2737E-13 2.2737E-13 2.2737E-13 2.2737E-13 2.2737E-13
50 6.1637E-03 6.1637E-03 5.1405E-03 2.8359E-03 -3.1985E-04 -3.6459E-03 -6.5574E-03 -8.7151E-03 -1.0080E-02 -1.0468E-02
48 3.9352E-03 3.9352E-03 3.2633E-03 1.9545E-03 3.0544E-04 -1.3614E-03 -2.8247E-03 -3.9875E-03 -6.8289E-03 -5.3456E-03
46 5.5038E-03 5.5038E-03 4.5401E-03 2.7119E-03 4.9782E-04 -1.6302E-03 -3.3906E-03 -4.6973E-03 -5.5706E-03 -6.0662E-03
44 7.2408E-03 7.2408E-03 5.9716E-03 3.6298E-03 8.7716E-04 -1.6998E-03 -3.8057E-03 -5.3889E-03 -6.4931E-03 -7.1448E-03
42 9.6240E-03 9.6240E-03 7.8992E-03 4.8486E-03 1.4004E-03 -1.7303E-03 -4.2573E-03 -6.1841E-03 -7.5809E-03 -8.4489E-03
40 1.2939E-02 1.2939E-02 1.0555E-02 6.5175E-03 2.1224E-03 -1.7806E-03 -4.9471E-03 -7.4452E-03 -9.3441E-03 -1.0584E-02
38 1.7444E-02 1.7444E-02 1.4141E-02 8.7786E-03 3.1433E-03 -1.7972E-03 -5.6876E-03 -9.2618E-03 -1.1944E-02 -1.3752E-02
36 2.3562E-02 2.3562E-02 1.8986E-02 1.1862E-02 4.5986E-03 -1.7673E-03 -7.2236E-03 -1.1941E-02 -1.5817E-02 -2.5407E-02
34 3.1867E-02 3.1867E-02 2.5543E-02 1.6095E-02 6.6882E-03 -1.6723E-03 -9.1616E-03 -1.5908E-02 -2.1554E-02 -3.5337E-02
32 4.3148E-02 4.3148E-02 3.4440E-02 2.1943E-02 9.6851E-03 -1.5193E-03 -1.2034E-02 -2.1790E-02 -2.9996E-02 -5.0172E-02
30 5.8479E-02 5.8479E-02 4.6524E-02 3.0042E-02 1.3940E-02 -1.3697E-03 -1.6345E-02 -3.0474E-02 -4.2313E-02 -7.1103E-02
28 7.9276E-02 7.9276E-02 6.2903E-02 4.1223E-02 1.9865E-02 -1.3909E-03 -2.2843E-02 -4.3190E-02 -6.8079E-02 -1.0073E-01
26 1.0734E-01 1.0734E-01 8.4937E-02 5.6498E-02 2.7874E-02 -1.9369E-03 -3.2598E-02 -6.1603E-02 -8.5409E-02 -1.0073E-01
24 1.4481E-01 1.4481E-01 1.1416E-01 7.6980E-02 3.8255E-02 -3.6703E-03 -4.7113E-02 -8.7934E-02 -1.2109E-01 -1.4222E-01
22 1.9402E-01 1.9402E-01 1.5210E-01 1.0371E-01 5.0953E-02 -7.7344E-03 -6.8469E-02 -1.2510E-01 -1.7073E-01 -1.9966E-01
20 2.5710E-01 2.5710E-01 1.9991E-01 1.3732E-01 6.5234E-02 -1.5970E-02 -9.9492E-02 -1.7685E-01 -2.3889E-01 -3.2895E-01
18 3.3525E-01 3.3525E-01 2.5795E-01 1.7755E-01 7.9306E-02 -3.1148E-02 -1.4392E-01 -2.4790E-01 -3.3114E-01 -4.5399E-01
16 4.2766E-01 4.2766E-01 3.2538E-01 2.2267E-01 9.0033E-02 -5.7156E-02 -2.0650E-01 -3.4391E-01 -4.5399E-01 -6.1445E-01
14 5.3032E-01 5.3032E-01 3.9969E-01 2.6913E-01 9.2947E-02 -9.9040E-02 -2.9293E-01 -4.7126E-01 -6.1445E-01 -8.1932E-01
12 6.3512E-01 6.3512E-01 4.7622E-01 3.1156E-01 8.2777E-02 -1.6271E-01 -4.0951E-01 -6.3652E-01 -8.1932E-01 -1.0741E-01
10 7.3009E-01 7.3009E-01 5.4725E-01 3.4332E-01 5.4505E-02 -2.5398E-01 -5.6244E-01 -8.4557E-01 -1.0741E-01 -1.3818E-01
8 8.0183E-01 8.0183E-01 6.0183E-01 3.5659E-01 3.6111E-03 -3.7701E-01 -9.9296E-01 -1.1024E+00 -1.3818E-01 -1.7417E-01
6 8.4144E-01 8.4144E-01 6.2904E-01 3.4253E-01 7.5060E-02 -5.3400E-01 -9.9296E-01 -1.1024E+00 -1.3818E-01 -1.7417E-01
4 8.5138E-01 8.5138E-01 6.2659E-01 2.9588E-01 -1.8766E-01 -7.2944E-01 -1.2732E+00 -1.7616E+00 -2.1517E+00 -2.4004E+00
2 8.4242E-01 8.4242E-01 6.0524E-01 2.2448E-01 3.2813E-01 -9.5365E-01 -1.5851E+00 -2.1508E+00 -2.6004E+00 -2.8865E+00
1 9.4053E-01 8.3848E-01 5.8972E-01 1.8552E-01 -4.0220E-01 -1.0682E+00 -1.7385E+00 -2.3383E+00 -2.8148E+00 -3.1184E+00

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```

ixi =      11      12      13      14      15      16      17      18      19      20
ieta
51 2.2737E-13 2.2737E-13 2.2737E-13 2.2737E-13 2.2737E-13 2.2737E-13 2.2737E-13 2.2737E-13 2.2737E-13 2.2737E-13
50 -1.0193E-02 -9.3111E-03 -7.9694E-03 -6.3378E-03 -4.6034E-03 -2.9598E-03 -1.5869E-03 -6.2181E-04 -1.1495E-04 1.7802E-05
48 -5.5242E-03 -5.3699E-03 -4.8306E-03 -4.0154E-03 -2.9866E-03 -1.9142E-03 -9.2088E-04 -1.7096E-04 2.4389E-04 3.5799E-04
46 -6.1360E-03 -5.8386E-03 -5.1692E-03 -4.1833E-03 -2.9835E-03 -1.7143E-03 -5.4932E-04 3.3978E-04 8.4368E-04 9.9247E-04

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44 -7.3253E-03 -7.0045E-03 -6.1671E-03 -4.9401E-03 -3.3952E-03 -1.7386E-03 -1.9351E-04 1.0138E-03 1.7272E-03 1.9607E-03
42 -8.7259E-03 -8.3449E-03 -7.3054E-03 -5.6995E-03 -3.7028E-03 -1.5515E-03 4.7735E-04 2.0987E-03 3.0975E-03 3.4559E-03
40 -1.1020E-02 -1.0536E-02 -9.1336E-03 -6.9543E-03 -4.2497E-03 -1.3369E-03 1.4292E-03 3.6829E-03 5.1269E-03 5.6899E-03
38 -1.4420E-02 -1.3770E-02 -1.1820E-02 -8.7923E-03 -5.0518E-03 -1.0330E-03 2.8010E-03 5.9801E-03 8.0964E-03 8.9879E-03
36 -1.9453E-02 -1.8532E-02 -1.5763E-02 -1.1490E-02 -6.2423E-03 -6.2299E-04 4.7520E-03 9.2747E-03 1.2395E-02 1.3808E-02
34 -2.6814E-02 -2.5464E-02 -2.1494E-02 -1.5423E-02 -8.0034E-03 -8.0952E-05 7.5059E-03 1.3962E-02 1.8564E-02 2.0793E-02
32 -3.7464E-02 -3.5477E-02 -2.9776E-02 -2.1135E-02 -1.0611E-02 6.1229E-04 1.1364E-02 2.0583E-02 2.7338E-02 3.0825E-02
30 -5.2815E-02 -4.9842E-02 -4.1684E-02 -2.9407E-02 -1.4474E-02 1.4531E-03 1.6715E-02 2.9856E-02 3.9699E-02 4.5089E-02
28 -7.4660E-02 -7.0315E-02 -5.8716E-02 -4.1337E-02 -2.0198E-02 2.3816E-03 2.4034E-02 4.2697E-02 5.6905E-02 6.5129E-02
26 -1.0552E-01 -9.9286E-02 -8.2922E-02 -5.8451E-02 -2.8665E-02 3.2177E-03 3.3840E-02 6.0217E-02 8.0499E-02 9.2867E-02
24 -1.4871E-01 -1.3993E-01 -1.1705E-01 -8.2839E-02 -4.1159E-02 3.5477E-03 4.6601E-02 8.3648E-02 1.1225E-01 1.3057E-01
22 -2.0851E-01 -1.9635E-01 -1.6470E-01 -1.1732E-01 -5.9542E-02 2.5402E-03 6.2544E-02 1.1417E-01 1.5396E-01 1.8067E-01
20 -2.9023E-01 -2.7366E-01 -2.3044E-01 -1.6562E-01 -8.6477E-02 -1.3181E-03 8.1339E-02 1.5259E-01 2.0717E-01 2.4550E-01
18 -4.0022E-01 -3.7802E-01 -3.1990E-01 -2.3253E-01 -1.2567E-01 -1.0551E-02 1.0167E-01 1.9890E-01 2.7260E-01 3.2674E-01
16 -5.4567E-01 -5.1647E-01 -4.3965E-01 -3.2392E-01 -1.8207E-01 -2.9015E-02 1.2074E-01 2.5162E-01 3.4948E-01 4.2479E-01
14 -7.3408E-01 -6.9643E-01 -5.9686E-01 -4.4653E-01 -2.6179E-01 -6.2067E-02 1.3388E-01 3.0732E-01 4.3490E-01 5.3806E-01
12 -9.7245E-01 -9.2496E-01 -7.9858E-01 -6.0735E-01 -3.7179E-01 -1.1639E-01 1.3450E-01 3.6019E-01 5.2329E-01 6.6251E-01
10 -1.2661E+00 -1.2075E+00 -1.0506E+00 -8.1263E-01 -5.1888E-01 -1.9932E-01 1.1468E-01 4.0228E-01 6.0652E-01 7.9143E-01
8 -1.6173E+00 -1.5466E+00 -1.3560E+00 -1.0665E+00 -7.0851E-01 -3.1761E-01 6.6339E-02 4.2434E-01 6.7465E-01 9.1588E-01
6 -2.0239E+00 -1.9405E+00 -1.7142E+00 -1.3698E+00 -9.4328E-01 -4.7598E-01 1.17139E-02 4.1731E-01 7.1732E-01 1.0256E+00
4 -2.4809E+00 -2.3840E+00 -2.1207E+00 -1.7196E+00 -1.2222E+00 -6.7590E-01 1.3935E-01 3.7416E-01 7.2548E-01 1.1104E+00
2 -2.9798E+00 -2.8702E+00 -2.5703E+00 -2.1124E+00 -1.5438E+00 -9.1801E-01 3.0254E-01 2.8938E-01 6.9190E-01 1.1606E+00
1 -3.2181E+00 -3.1035E+00 -2.7878E+00 -2.3049E+00 -1.7043E+00 -1.0428E+00 3.9124E-01 2.3459E-01 6.6435E-01 1.1537E+00

```

ixl = 21

leta

```

51 -4.4916E-05
50 1.7802E-05
48 3.5799E-04
46 9.9247E-04
44 1.9607E-03
42 3.4559E-03
40 5.6899E-03
38 8.9879E-03
36 1.3808E-02
34 2.0793E-02
32 3.0825E-02
30 4.5089E-02
28 6.5129E-02
26 9.2867E-02
24 1.3057E-01
22 1.8067E-01
20 2.4550E-01
18 3.2674E-01
16 4.2479E-01
14 5.3806E-01
12 6.6251E-01
10 7.9143E-01
8 9.1588E-01
6 1.0256E+00
4 1.1104E+00
2 1.1606E+00
1 1.3839E+00

```

*** Converged solution at time level 210

Convergence history for continuity equation

level	chgmax	chgavg	*** Without artificial viscosity ***				*** With artificial viscosity ***			
			resl2	resavg	resmax	resmax loc.	resl2	resavg	resmax	resmax loc.
1	0.000E+00	0.000E+00	1.391E-10	2.561E-12	2.354E-11	(5, 46)	1.046E-09	1.946E-11	1.157E-10	(20, 50)
2	0.000E+00	0.000E+00	1.125E+02	1.725E+00	2.180E+01	(2, 2)	1.123E+02	1.727E+00	2.153E+01	(2, 2)
10	1.174E-02	0.000E+00	9.439E+01	1.524E+00	2.402E+01	(18, 50)	6.449E+01	1.367E+00	1.181E+01	(2, 50)
20	5.565E-03	0.000E+00	4.700E+01	6.519E-01	9.234E+00	(8, 49)	3.148E+01	5.232E-01	4.983E+00	(9, 50)
30	8.877E-04	0.000E+00	1.978E+01	2.374E-01	5.950E+00	(10, 50)	9.696E+00	1.735E-01	2.017E+00	(17, 50)
40	4.418E-04	0.000E+00	8.325E+00	9.466E-02	3.261E+00	(2, 50)	1.554E+00	3.352E-02	3.134E-01	(3, 50)
50	4.900E-04	0.000E+00	9.727E+00	9.824E-02	2.883E+00	(10, 50)	9.155E-01	1.821E-02	1.584E-01	(5, 50)
60	4.596E-04	0.000E+00	8.981E+00	8.614E-02	2.722E+00	(10, 50)	5.594E-01	1.045E-02	8.591E-02	(2, 13)
70	3.800E-04	0.000E+00	8.769E+00	8.273E-02	2.634E+00	(10, 50)	3.277E-01	6.398E-03	5.333E-02	(2, 11)
80	1.648E-04	0.000E+00	8.397E+00	7.931E-02	2.539E+00	(10, 50)	1.888E-01	3.460E-03	3.817E-02	(3, 50)
90	7.953E-05	0.000E+00	8.094E+00	7.750E-02	2.469E+00	(10, 50)	1.051E-01	1.729E-03	2.931E-02	(3, 50)
100	3.961E-05	0.000E+00	7.971E+00	7.683E-02	2.450E+00	(10, 50)	4.537E-02	7.905E-04	1.145E-02	(3, 50)
110	2.001E-05	0.000E+00	7.948E+00	7.673E-02	2.448E+00	(10, 50)	2.194E-02	4.136E-04	3.946E-03	(2, 9)
120	1.015E-05	0.000E+00	7.940E+00	7.673E-02	2.445E+00	(10, 50)	1.196E-02	2.221E-04	2.051E-03	(3, 44)
130	5.181E-06	0.000E+00	7.925E+00	7.667E-02	2.441E+00	(10, 50)	7.243E-03	1.232E-04	1.837E-03	(3, 50)
140	2.530E-06	0.000E+00	7.915E+00	7.663E-02	2.439E+00	(10, 50)	3.405E-03	5.734E-05	9.701E-04	(3, 50)
150	1.217E-06	0.000E+00	7.912E+00	7.661E-02	2.438E+00	(10, 50)	1.438E-03	2.641E-05	2.876E-04	(3, 50)
160	6.252E-07	0.000E+00	7.911E+00	7.660E-02	2.438E+00	(10, 50)	7.458E-04	1.350E-05	1.641E-04	(3, 50)
170	3.156E-07	0.000E+00	7.910E+00	7.660E-02	2.438E+00	(10, 50)	4.039E-04	6.727E-06	1.144E-04	(3, 50)
180	1.477E-07	0.000E+00	7.909E+00	7.660E-02	2.438E+00	(10, 50)	1.728E-04	3.027E-06	4.309E-05	(3, 50)
190	7.415E-08	0.000E+00	7.909E+00	7.660E-02	2.438E+00	(10, 50)	8.463E-05	1.622E-06	1.491E-05	(3, 50)
200	3.853E-08	0.000E+00	7.909E+00	7.659E-02	2.438E+00	(10, 50)	5.003E-05	9.401E-07	1.111E-05	(3, 50)

Convergence history for x-momentum equation

level	chgmax	chgavg	*** Without artificial viscosity ***				*** With artificial viscosity ***			
			resl2	resavg	resmax	resmax loc.	resl2	resavg	resmax	resmax loc.
1	0.000E+00	0.000E+00	2.838E-09	5.347E-11	5.487E-10	(18, 50)	3.008E-09	5.676E-11	4.508E-10	(18, 50)
2	0.000E+00	0.000E+00	4.804E+02	7.178E+00	8.874E+01	(2, 3)	4.770E+02	7.137E+00	8.547E+01	(2, 3)
10	5.413E-02	0.000E+00	3.268E+02	5.660E+00	9.730E+01	(2, 49)	2.632E+02	5.375E+00	6.270E+01	(2, 50)
20	1.326E-02	0.000E+00	1.550E+02	2.484E+00	4.188E+01	(5, 49)	1.055E+02	1.756E+00	2.392E+01	(3, 50)
30	1.717E-02	0.000E+00	6.776E+01	1.302E+01	1.265E+01	(7, 50)	3.087E+01	6.419E-01	5.175E+00	(20, 50)
40	1.318E-02	0.000E+00	4.617E+01	8.237E-01	1.624E+01	(2, 50)	1.902E+01	4.082E-01	2.223E+00	(7, 49)
50	8.716E-03	0.000E+00	3.414E+01	5.833E-01	1.321E+01	(2, 50)	1.187E+01	2.266E-01	1.666E+00	(2, 20)
60	4.966E-03	0.000E+00	2.918E+01	4.504E-01	1.226E+01	(2, 50)	6.928E+00	1.301E-01	1.062E+00	(2, 17)
70	2.967E-03	0.000E+00	2.760E+01	4.079E-01	1.161E+01	(2, 50)	3.630E+00	6.774E-02	5.766E-01	(2, 14)
80	1.581E-03	0.000E+00	2.667E+01	4.071E-01	1.054E+01	(2, 50)	1.814E+00	3.477E-02	2.785E-01	(2, 13)
90	7.426E-04	0.000E+00	2.606E+01	4.135E-01	9.719E+00	(2, 50)	9.171E-01	1.750E-02	1.378E-01	(2, 50)
100	3.712E-04	0.000E+00	2.587E+01	4.168E-01	9.372E+00	(2, 50)	4.641E-01	9.083E-03	6.773E-02	(2, 11)
110	1.851E-04	0.000E+00	2.595E+01	4.193E-01	9.332E+00	(2, 50)	2.359E-01	4.555E-03	3.347E-02	(2, 11)
120	9.473E-05	0.000E+00	2.600E+01	4.211E-01	9.297E+00	(2, 50)	1.208E-01	2.285E-03	1.737E-02	(6, 39)
130	4.751E-05	0.000E+00	2.599E+01	4.218E-01	9.249E+00	(2, 50)	6.112E-02	1.164E-03	8.743E-03	(2, 50)
140	2.270E-05	0.000E+00	2.598E+01	4.220E-01	9.220E+00	(2, 50)	2.877E-02	5.508E-04	4.373E-03	(2, 50)
150	1.140E-05	0.000E+00	2.598E+01	4.222E-01	9.212E+00	(2, 50)	1.382E-02	2.615E-04	2.037E-03	(2, 11)

Laminar Prandtl number, prlr = 5.0306E+04
 "Reference" Prandtl number, prr = 8.0489E+02
 Specific heat at const. press. = 6.0060E+03 ft2/sec2-deg R
 Specific heat at const. vol. = 4.2900E+03 ft2/sec2-deg R
 Gas constant, rg = 1.7160E+03 ft2/sec2-deg R
 Pressure, pr = 2.1162E+03 lbf/ft2
 Stagnation enthalpy, hstagr = 3.1421E+06 ft2/sec2

Boundary condition parameters

```

jbc2(1,2) = -1      gbc2(1,2) = 0.0000E+00
jbc2(2,2) = -1      gbc2(2,2) = 0.0000E+00
jbc2(3,2) = -1      gbc2(3,2) = 0.0000E+00

jbc1(1,1) = 42      gbc1(1,1) = 0.0000E+00      jbc1(1,2) = 42      gbc1(1,2) = 0.0000E+00
jbc1(2,1) = 12      gbc1(2,1) = 0.0000E+00      jbc1(2,2) = 12      gbc1(2,2) = 0.0000E+00
jbc1(3,1) = 21      gbc1(3,1) = 0.0000E+00      jbc1(3,2) = 21      gbc1(3,2) = 0.0000E+00

jbc2(1,1) = 42      gbc2(1,1) = 0.0000E+00
jbc2(2,1) = 11      gbc2(2,1) = 0.0000E+00
jbc2(3,1) = 21      gbc2(3,1) = 0.0000E+00
  
```

x-velocity at time level 360

ix1 =	1	2	4	6	8	10	12	14	16	18
leta										
51	9.6267E-01	9.6610E-01	9.7765E-01	1.0003E+00	1.0142E+00	1.0151E+00	1.0113E+00	1.0001E+00	1.0003E+00	1.0006E+00
50	9.6610E-01	9.6610E-01	9.7765E-01	1.0003E+00	1.0142E+00	1.0151E+00	1.0113E+00	1.0001E+00	1.0003E+00	1.0006E+00
48	9.4739E-01	9.4739E-01	9.6495E-01	9.9680E-01	1.0152E+00	1.0174E+00	1.0168E+00	1.0178E+00	1.0180E+00	1.0175E+00
46	9.1498E-01	9.1498E-01	9.4529E-01	9.9579E-01	1.0205E+00	1.0209E+00	1.0198E+00	1.0224E+00	1.0242E+00	1.0241E+00
44	8.6060E-01	8.6060E-01	9.1280E-01	9.9483E-01	1.0305E+00	1.0282E+00	1.0257E+00	1.0290E+00	1.0312E+00	1.0314E+00
42	7.7663E-01	7.7663E-01	8.6040E-01	9.8977E-01	1.0455E+00	1.0421E+00	1.0375E+00	1.0405E+00	1.0425E+00	1.0421E+00
40	6.5801E-01	6.5801E-01	7.7921E-01	9.7073E-01	1.0625E+00	1.0649E+00	1.0569E+00	1.0582E+00	1.0593E+00	1.0577E+00
38	5.0790E-01	5.0790E-01	6.6120E-01	9.1947E-01	1.0702E+00	1.0962E+00	1.0871E+00	1.0835E+00	1.0821E+00	1.0787E+00
36	3.4250E-01	3.4250E-01	5.0810E-01	8.1517E-01	1.0428E+00	1.1237E+00	1.1271E+00	1.1180E+00	1.1117E+00	1.1052E+00
34	1.8780E-01	1.8780E-01	3.3899E-01	6.5181E-01	9.4608E-01	1.1127E+00	1.1622E+00	1.1600E+00	1.1485E+00	1.1372E+00
32	6.7106E-02	6.7106E-02	1.8537E-01	4.5544E-01	7.6781E-01	1.0170E+00	1.1514E+00	1.1914E+00	1.1884E+00	1.1744E+00
30	-1.1141E-02	-1.1141E-02	7.1475E-02	2.7325E-01	5.4509E-01	8.2377E-01	1.0431E+00	1.1663E+00	1.2088E+00	1.2096E+00
28	-5.2952E-02	-5.2952E-02	1.2295E-01	1.3791E-01	3.4018E-01	5.8440E-01	8.3294E-01	1.0355E+00	1.1599E+00	1.2131E+00
26	-6.9252E-02	-6.9252E-02	3.4973E-02	5.2467E-02	1.8883E-01	3.6983E-01	5.8581E-01	8.1011E-01	1.0026E+00	1.1330E+00
24	-6.9392E-02	-6.9392E-02	-4.8606E-02	4.7725E-03	9.1064E-02	2.1261E-01	3.7222E-01	5.6459E-01	7.6867E-01	9.5247E-01
22	-6.0291E-02	-6.0291E-02	-4.8654E-02	-1.8184E-02	3.3154E-02	1.0960E-01	2.1747E-01	3.6070E-01	5.3400E-01	7.1972E-01
20	-4.7171E-02	-4.7171E-02	-4.1662E-02	-2.6257E-02	1.7653E-03	4.6946E-02	1.1587E-01	2.1494E-01	3.4607E-01	5.0290E-01
18	-3.3786E-02	-3.3786E-02	-3.2198E-02	-2.6257E-02	-1.3056E-02	1.1724E-02	5.3826E-02	1.1951E-01	2.1301E-01	3.3371E-01
16	-2.2362E-02	-2.2362E-02	-2.3054E-02	-2.2579E-02	-1.8107E-02	-5.1673E-03	1.8733E-02	6.1071E-02	1.2551E-01	2.1381E-01
14	-1.3750E-02	-1.3750E-02	-1.5544E-02	-1.7787E-02	-1.7954E-02	-1.2967E-02	8.5216E-04	2.7556E-02	7.0953E-02	1.3348E-01
12	-7.8610E-03	-7.8610E-03	-9.9725E-03	-1.3170E-02	-1.5364E-02	-1.4130E-02	6.7407E-03	9.7728E-03	3.8398E-02	8.1487E-02
10	-4.0507E-03	-4.0507E-03	-6.1114E-03	-9.2435E-03	-1.1937E-02	-1.2381E-02	-8.6465E-03	1.3338E-03	1.9740E-02	4.0542E-02
8	-1.9891E-03	-1.9891E-03	-5.5567E-03	-6.1124E-03	-8.5082E-03	-9.4741E-03	-7.7231E-03	-1.8989E-03	9.4937E-03	2.7901E-02
6	-8.2689E-04	-8.2689E-04	-1.9158E-03	-3.7040E-03	-5.4564E-03	-6.3574E-03	-5.6243E-03	-2.4454E-03	4.1147E-03	1.5009E-02
4	-2.6577E-04	-2.6577E-04	-8.7927E-04	-1.8889E-03	-2.9023E-03	-3.4882E-03	-3.2629E-03	-1.7536E-03	1.4629E-03	6.9262E-03
2	-4.2072E-05	-4.2072E-05	-2.2887E-04	-5.3662E-04	-8.5011E-04	-1.0430E-03	-1.0003E-03	-6.0581E-04	2.7960E-04	1.8107E-03
1	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00

ix1 =	20	22	24	26	28	30	32	34	36	38
leta										
51	1.0008E+00	1.0010E+00	1.0011E+00	1.0011E+00	1.0011E+00	1.0010E+00	1.0008E+00	1.0006E+00	1.0003E+00	1.0001E+00
50	1.0069E+00	1.0066E+00	1.0066E+00	1.0065E+00	1.0063E+00	1.0061E+00	1.0059E+00	1.0056E+00	1.0053E+00	1.0051E+00
48	1.0165E+00	1.0153E+00	1.0153E+00	1.0152E+00	1.0150E+00	1.0148E+00	1.0146E+00	1.0143E+00	1.0140E+00	1.0138E+00
46	1.0232E+00	1.0216E+00	1.0216E+00	1.0215E+00	1.0213E+00	1.0211E+00	1.0209E+00	1.0206E+00	1.0203E+00	1.0201E+00
44	1.0303E+00	1.0285E+00	1.0285E+00	1.0284E+00	1.0282E+00	1.0280E+00	1.0278E+00	1.0275E+00	1.0272E+00	1.0270E+00
42	1.0403E+00	1.0374E+00	1.0374E+00	1.0373E+00	1.0371E+00	1.0369E+00	1.0367E+00	1.0364E+00	1.0361E+00	1.0359E+00
40	1.0544E+00	1.0505E+00	1.0505E+00	1.0504E+00	1.0502E+00	1.0500E+00	1.0498E+00	1.0495E+00	1.0492E+00	1.0490E+00
38	1.0732E+00	1.0682E+00	1.0682E+00	1.0681E+00	1.0679E+00	1.0677E+00	1.0675E+00	1.0672E+00	1.0669E+00	1.0667E+00
36	1.0960E+00	1.0898E+00	1.0898E+00	1.0897E+00	1.0895E+00	1.0893E+00	1.0891E+00	1.0888E+00	1.0885E+00	1.0883E+00
34	1.1252E+00	1.1177E+00	1.1177E+00	1.1175E+00	1.1173E+00	1.1171E+00	1.1169E+00	1.1166E+00	1.1163E+00	1.1161E+00
32	1.1583E+00	1.1499E+00	1.1499E+00	1.1497E+00	1.1495E+00	1.1493E+00	1.1491E+00	1.1488E+00	1.1485E+00	1.1483E+00
30	1.1949E+00	1.1748E+00	1.1748E+00	1.1746E+00	1.1744E+00	1.1742E+00	1.1740E+00	1.1737E+00	1.1734E+00	1.1732E+00
28	1.2222E+00	1.2047E+00	1.2047E+00	1.2045E+00	1.2043E+00	1.2041E+00	1.2039E+00	1.2036E+00	1.2033E+00	1.2031E+00
26	1.2021E+00	1.2237E+00	1.2237E+00	1.2235E+00	1.2233E+00	1.2231E+00	1.2229E+00	1.2226E+00	1.2223E+00	1.2221E+00
24	1.0898E+00	1.1726E+00	1.1726E+00	1.1724E+00	1.1722E+00	1.1720E+00	1.1718E+00	1.1715E+00	1.1712E+00	1.1710E+00
22	8.9253E-01	1.0301E+00	1.1209E+00	1.1648E+00	1.1677E+00	1.1377E+00	1.0823E+00	1.0087E+00	9.2316E-01	8.3199E-01
20	6.7017E-01	8.2764E-01	9.5654E-01	1.0447E+00	1.0878E+00	1.0879E+00	1.0509E+00	9.8424E-01	9.6564E-01	9.3648E-01
18	4.7435E-01	6.2151E-01	7.5847E-01	8.6940E-01	9.4295E-01	9.7377E-01	9.6231E-01	9.1348E-01	8.3516E-01	7.3697E-01
16	3.2334E-01	4.4627E-01	5.7044E-01	6.8163E-01	7.6673E-01	8.1634E-01	8.2625E-01	7.9745E-01	7.3536E-01	6.4851E-01
14	2.1482E-01	3.1078E-01	4.1320E-01	5.1099E-01	5.9251E-01	6.4706E-01	6.6876E-01	6.5560E-01	6.1035E-01	5.3949E-01
12	1.3971E-01	2.1101E-01	2.9012E-01	3.6897E-01	4.3809E-01	4.8485E-01	5.1338E-01	5.0979E-01	4.7850E-01	4.2425E-01
10	8.671E-02	1.3924E-01	1.9696E-01	2.5625E-01	3.1005E-01	3.5119E-01	3.7391E-01	3.7500E-01	3.5437E-01	3.1491E-01
8	5.4209E-02	8.8104E-02	1.2762E-01	1.6908E-01	2.0760E-01	2.3800E-01	2.5587E-01	2.5852E-01	2.4550E-01	2.1856E-01
6	3.0898E-02	5.1726E-02	7.6390E-02	1.0267E-01	1.2750E-01	1.4750E-01	1.5973E-01	1.6227E-01	1.5464E-01	1.3785E-01
4	1.5024E-02	2.5779E-02	3.8668E-02	5.2557E-02	6.5832E-02	7.6682E-02	8.3484E-02	8.5150E-02	8.1353E-02	7.2582E-02
2	4.1083E-03	7.1910E-03	1.0917E-02	1.4966E-02	1.8869E-02	2.2091E-02	2.4145E-02	2.4699E-02	2.3642E-02	2.1107E-02
1	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00

ix1 =	40	42	44	46	48	50	51
leta							
51	9.9979E-01	9.9953E-01	9.9929E-01	9.9910E-01	9.9897E-01	9.9890E-01	9.9414E-01
50	9.9629E-01	9.9513E-01	9.9417E-01	9.9343E-01	9.9295E-01	9.9278E-01	9.9278E-01
48	9.9686E-01	9.9515E-01	9.9375E-01	9.9265E-01	9.9200E-01	9.9179E-01	9.9179E-01
46	9.9638E-01	9.9416E-01	9.9235E-01	9.9102E-01	9.9023E-01	9.9000E-01	9.9000E-01
44	9.9391E-01	9.9086E-01	9.8838E-01	9.8656E-01	9.8550E-01	9.8518E-01	9.8518E-01
42	9.9021E-01	9.8607E-01	9.8270E-01	9.8023E-01	9.7877E-01	9.7831E-01	9.7831E-01
40	9.8437E-01	9.7870E-01	9.7409E-01	9.7069E-01	9.6844E-01	9.6798E-01	9.6798E-01
38	9.7566E-01	9.6784E-01	9.6146E-01	9.5671E-01	9.5382E-01	9.5284E-01	9.5284E-01
36	9.6318E-01	9.5235E-01	9.4346E-01	9.3681E-01	9.3249E-01	9.3124E-01	9.3124E-01
34	9.4800E-01	9.3100E-01	9.1860E-01	9.0925E-01	9.0337E-01	9.0122E-01	9.0122E-01
32	9.2341E-01	9.0245E-01	8.8539E-01	8.7224E-01	8.6388E-01	8.6071E-01	8.6071E-01
30	8.9509E-01	8.6656E-01	8.4263E-01	8.2429E-01	8.1245E-01	8.0783E-01	8.0783E-01
28	8.6142E-01	8.2261E-01	7.8979E-01	7.6446E-01	7.4794E-01	7.4132E-01	7.4132E-01
26	8.2356E-01	7.7166E-01	7.2740E-01	6.9300E-01	6.7040E-01	6.6114E-01	6.6114E-01
24	7.8330E-01	7.1554E-01	6.5725E-01	6.1164E-01	5.8144E-01	5.6887E-01	5.6887E-01
22	7.4109E-01	6.5604E-01	5.8199E-01	5.2348E-01	4.8440E-01	4.6786E-01	4.6786E-01
20	6.9247E-01	5.9250E-01	5.0378E-01	4.3261E-01	3.8440E-01	3.6360E-01	3.6360E-01
18	6.2913E-01	5.2167E-01	4.2385E-01	3.4383E-01	2.8856E-01	2.6407E-01	2.6407E-01
16	5.4721E-01	4.4230E-01	3.4420E-01	2.6224E-01	2.0445E-01	1.7804E-01	1.7804E-01
14	4.5207E-01	3.5831E-01	2.6849E-01	1.9205E-01	1.3713E-01	1.1121E-01	1.1121E-01
12	3.5395E-01	2.7639E-01	2.0064				

ix1 =	1	2	4	6	8	10	12	14	16	18
beta										
51	3.1342E-02	2.1952E-03	2.0504E-03	1.7918E-03	1.4097E-03	9.3822E-04	4.0870E-04	-1.4077E-04	-6.8794E-04	-1.1920E-03
50	4.9751E-02	4.9751E-02	3.8739E-02	2.3334E-02	9.3175E-03	-3.9895E-03	-1.6026E-02	-1.1178E-02	-7.1565E-03	-6.5283E-03
48	3.0001E-02	3.0001E-02	2.3861E-02	1.8691E-02	9.7681E-03	-6.5430E-05	-4.1677E-03	-9.9676E-03	-1.3437E-02	-1.4008E-02

46	2.5651E-02	2.5651E-02	2.0005E-02	1.6516E-02	5.9261E-03	-7.0203E-03	-1.4620E-02	-1.8600E-02	-2.1611E-02	-2.3033E-02
44	2.1240E-02	2.1240E-02	1.4995E-02	1.0992E-02	-4.1048E-03	-2.1266E-02	-3.0326E-02	-3.4493E-02	-3.7193E-02	-3.7933E-02
42	4.5056E-03	4.5056E-03	-1.7540E-03	-5.9929E-03	-2.4812E-02	-4.5085E-02	-5.4872E-02	-5.8912E-02	-6.0976E-02	-6.0387E-02
40	-3.0853E-02	-3.0853E-02	-3.7323E-02	-4.2153E-02	-6.2384E-02	-8.3802E-02	-9.3540E-02	-9.6260E-02	-9.6110E-02	-9.2792E-02
38	-9.6693E-02	-9.6693E-02	-1.0333E-01	-1.0813E-01	-1.2462E-01	-1.4256E-01	-1.5010E-01	-1.4960E-01	-1.4518E-01	-1.3733E-01
36	-1.9747E-01	-1.9747E-01	-2.0412E-01	-2.0850E-01	-2.1661E-01	-2.2508E-01	-2.2660E-01	-2.2048E-01	-2.0964E-01	-1.9535E-01
34	-3.2054E-01	-3.2054E-01	-3.2706E-01	-3.3224E-01	-3.3285E-01	-3.2933E-01	-3.2112E-01	-3.0727E-01	-2.8877E-01	-2.6688E-01
32	-4.3946E-01	-4.3946E-01	-4.4557E-01	-4.5293E-01	-4.5253E-01	-4.4267E-01	-4.2609E-01	-4.0477E-01	-3.7937E-01	-3.5051E-01
30	-5.3337E-01	-5.3337E-01	-5.3861E-01	-5.4741E-01	-5.5079E-01	-5.4401E-01	-5.2748E-01	-5.0410E-01	-4.7589E-01	-4.4344E-01
28	-5.9800E-01	-5.9800E-01	-6.0227E-01	-6.1093E-01	-6.1756E-01	-6.1773E-01	-6.0949E-01	-5.9311E-01	-5.6999E-01	-5.4076E-01
26	-6.3896E-01	-6.3896E-01	-6.4268E-01	-6.5075E-01	-6.5889E-01	-6.6418E-01	-6.6530E-01	-6.6129E-01	-6.5119E-01	-6.3392E-01
24	-6.6330E-01	-6.6330E-01	-6.6699E-01	-6.7504E-01	-6.8335E-01	-6.9297E-01	-7.0067E-01	-7.0757E-01	-7.1251E-01	-7.1267E-01
22	-6.7658E-01	-6.7658E-01	-6.8070E-01	-6.8947E-01	-7.0050E-01	-7.1226E-01	-7.2491E-01	-7.3945E-01	-7.5598E-01	-7.7219E-01
20	-6.8274E-01	-6.8274E-01	-6.8763E-01	-6.9769E-01	-7.1099E-01	-7.2622E-01	-7.4350E-01	-7.6399E-01	-7.8850E-01	-8.1592E-01
18	-6.8445E-01	-6.8445E-01	-6.9031E-01	-7.0197E-01	-7.1780E-01	-7.3660E-01	-7.5836E-01	-7.8407E-01	-8.1464E-01	-8.4956E-01
16	-6.8348E-01	-6.8348E-01	-6.9039E-01	-7.0373E-01	-7.2203E-01	-7.4417E-01	-7.7001E-01	-8.0032E-01	-8.3590E-01	-8.7635E-01
14	-6.8105E-01	-6.8105E-01	-6.8901E-01	-7.0391E-01	-7.2444E-01	-7.4947E-01	-7.7877E-01	-8.1298E-01	-8.5277E-01	-8.9763E-01
12	-6.7800E-01	-6.7800E-01	-6.8692E-01	-7.0321E-01	-7.2564E-01	-7.5303E-01	-7.8512E-01	-8.2249E-01	-8.6574E-01	-9.1424E-01
10	-6.7491E-01	-6.7491E-01	-6.8464E-01	-7.0209E-01	-7.2609E-01	-7.5537E-01	-7.8963E-01	-8.2947E-01	-8.7548E-01	-9.2696E-01
8	-6.7207E-01	-6.7207E-01	-6.8247E-01	-7.0088E-01	-7.2613E-01	-7.5689E-01	-7.9279E-01	-8.3451E-01	-8.8270E-01	-9.3659E-01
6	-6.6963E-01	-6.6963E-01	-6.8057E-01	-6.9973E-01	-7.2599E-01	-7.5787E-01	-7.9501E-01	-8.3816E-01	-8.8803E-01	-9.4384E-01
4	-6.6759E-01	-6.6759E-01	-6.7895E-01	-6.9872E-01	-7.2577E-01	-7.5852E-01	-7.9660E-01	-8.4083E-01	-8.9200E-01	-9.4935E-01
2	-6.6584E-01	-6.6584E-01	-6.7754E-01	-6.9781E-01	-7.2552E-01	-7.5899E-01	-7.9783E-01	-8.4291E-01	-8.9516E-01	-9.5378E-01
1	-6.6255E-01	-6.6255E-01	-6.7584E-01	-6.9781E-01	-7.2552E-01	-7.5899E-01	-7.9783E-01	-8.4291E-01	-8.9516E-01	-9.5378E-01

ixl =	20	22	24	26	28	30	32	34	36	38
ieta										
51	-1.6212E-03	-1.9486E-03	-2.1536E-03	-2.2235E-03	-2.1536E-03	-1.9486E-03	-1.6212E-03	-1.1920E-03	-6.6794E-04	-1.4077E-04
50	-5.5059E-02	-3.1127E-03	5.9640E-04	5.3046E-03	1.0676E-02	1.6393E-02	2.2162E-02	2.7740E-02	3.2936E-02	3.7612E-02
48	-1.3322E-02	-1.1383E-02	-8.3231E-03	-4.2553E-03	6.4334E-04	6.1399E-02	1.1983E-02	1.7925E-02	2.3733E-02	2.9201E-02
46	-2.2404E-02	-2.0102E-02	-1.6510E-02	-1.1855E-02	-6.3712E-03	-3.1446E-04	6.0552E-03	1.2490E-02	1.8750E-02	2.4652E-02
44	-3.6276E-02	-3.2591E-02	-2.7238E-02	-2.0505E-02	-1.2719E-02	-4.2281E-03	4.6243E-03	1.3509E-02	2.2120E-02	3.0186E-02
42	-5.6908E-02	-5.1046E-02	-4.3203E-02	-3.3724E-02	-2.3086E-02	-1.1646E-02	4.8646E-04	1.2437E-02	2.4000E-02	3.4840E-02
40	-8.6202E-02	-7.6851E-02	-6.5180E-02	-5.1605E-02	-3.6592E-02	-2.0633E-02	4.2363E-03	1.2092E-02	2.7863E-02	4.2626E-02
38	-1.2602E-01	-1.1167E-01	-9.4684E-02	-7.5515E-02	-5.4644E-02	-3.2691E-02	-1.0205E-02	1.2163E-02	3.3782E-02	5.4052E-02
36	-1.7770E-01	-1.5685E-01	-1.3308E-01	-1.0678E-01	-7.8621E-02	-4.8626E-02	-1.8109E-02	1.2337E-02	4.1881E-02	6.9702E-02
34	-2.4184E-01	-2.1351E-01	-1.8190E-01	-1.4717E-01	-1.0974E-01	-7.0229E-02	-2.9481E-02	1.1494E-02	5.1574E-02	8.9602E-02
32	-3.1840E-01	-2.8271E-01	-2.4303E-01	-1.9924E-01	-1.5154E-01	-1.0054E-01	-4.7227E-02	7.0925E-03	6.0874E-02	1.1245E-01
30	-4.0679E-01	-3.6552E-01	-3.1889E-01	-2.6639E-01	-2.0795E-01	-1.4408E-01	-7.5937E-02	-5.2308E-03	6.5901E-02	1.3506E-01
28	-5.0513E-01	-4.6245E-01	-4.1181E-01	-3.5238E-01	-2.8378E-01	-2.0641E-01	-1.2160E-01	-3.1549E-02	6.0801E-02	1.5203E-01
26	-6.0784E-01	-5.7109E-01	-5.2209E-01	-4.5964E-01	-3.8312E-01	-2.9286E-01	-1.9042E-01	-7.8653E-02	5.8492E-02	1.5625E-01
24	-7.0411E-01	-6.8244E-01	-6.4377E-01	-5.8538E-01	-5.0593E-01	-4.0559E-01	-2.8640E-01	-1.5193E-01	-7.4925E-03	1.4043E-01
22	-7.8313E-01	-7.8207E-01	-7.6190E-01	-7.1661E-01	-6.4245E-01	-5.3847E-01	-4.0654E-01	-2.5125E-01	-7.9483E-02	1.0015E-01
20	-8.4213E-01	-8.5995E-01	-8.6012E-01	-8.3330E-01	-7.7210E-01	-6.7210E-01	-5.3553E-01	-3.6529E-01	-1.7020E-01	3.8875E-02
18	-8.8569E-01	-9.1651E-01	-9.3245E-01	-9.2234E-01	-8.7582E-01	-7.8581E-01	-6.5024E-01	-4.7259E-01	-2.6146E-01	-2.9292E-02
16	-9.1891E-01	-9.5744E-01	-9.8327E-01	-9.8416E-01	-9.4848E-01	-8.6944E-01	-7.3517E-01	-5.5502E-01	-3.3454E-01	-8.6850E-02
14	-9.4481E-01	-9.8848E-01	-1.0195E+00	-1.0262E+00	-9.9635E-01	-9.1947E-01	-7.8991E-01	-6.0834E-01	-3.8214E-01	-1.2455E-01
12	-9.6504E-01	-1.0121E+00	-1.0463E+00	-1.0558E+00	-1.0281E+00	-9.5252E-01	-8.2278E-01	-6.3901E-01	-4.0826E-01	-1.4375E-01
10	-9.8074E-01	-1.0305E+00	-1.0668E+00	-1.0776E+00	-1.0503E+00	-9.7411E-01	-8.4263E-01	-6.5587E-01	-4.2085E-01	-1.5090E-01
8	-9.9282E-01	-1.0448E+00	-1.0827E+00	-1.0942E+00	-1.0667E+00	-9.8926E-01	-8.5550E-01	-6.6554E-01	-4.2652E-01	-1.5201E-01
6	-1.0021E+00	-1.0559E+00	-1.0951E+00	-1.1073E+00	-1.0794E+00	-1.0007E+00	-8.6470E-01	-6.7177E-01	-4.2942E-01	-1.5086E-01
4	-1.0092E+00	-1.0645E+00	-1.1049E+00	-1.1176E+00	-1.0896E+00	-1.0097E+00	-8.7188E-01	-6.7642E-01	-4.3089E-01	-1.4927E-01
2	-1.0150E+00	-1.0717E+00	-1.1131E+00	-1.1264E+00	-1.0982E+00	-1.0175E+00	-8.7805E-01	-6.8042E-01	-4.3229E-01	-1.4785E-01
1	-1.0150E+00	-1.0717E+00	-1.1131E+00	-1.1264E+00	-1.0982E+00	-1.0175E+00	-8.7805E-01	-6.8042E-01	-4.3229E-01	-1.4785E-01

ixl =	40	42	44	46	48	50	51
ieta							
51	4.1517E-04	9.4494E-04	1.4153E-03	1.7966E-03	2.0649E-03	2.2035E-03	2.5053E-02
50	4.1677E-02	4.5073E-02	4.7762E-02	4.9710E-02	5.0870E-02	5.1218E-02	5.1218E-02
48	3.4152E-02	3.8443E-02	4.1967E-02	4.4642E-02	4.6438E-02	4.7280E-02	4.7280E-02
46	2.9995E-02	3.4641E-02	3.8481E-02	4.1427E-02	4.3427E-02	4.4419E-02	4.4419E-02
44	3.7477E-02	4.3802E-02	4.9017E-02	5.3006E-02	5.5683E-02	5.7021E-02	5.7021E-02
42	4.4642E-02	5.3157E-02	6.0184E-02	6.5572E-02	6.9180E-02	7.1045E-02	7.1045E-02
40	5.5981E-02	6.7589E-02	7.7173E-02	8.4528E-02	8.9465E-02	9.2041E-02	9.2041E-02
38	7.2424E-02	8.8426E-02	1.0166E-01	1.1184E-01	1.1848E-01	1.2228E-01	1.2228E-01
36	9.5027E-02	1.1717E-01	1.3555E-01	1.4970E-01	1.5926E-01	1.6431E-01	1.6431E-01
34	1.2445E-01	1.5511E-01	1.8067E-01	2.0042E-01	2.1382E-01	2.2090E-01	2.2090E-01
32	1.6017E-01	2.0246E-01	2.3794E-01	2.6548E-01	2.8427E-01	2.9416E-01	2.9416E-01
30	1.9977E-01	2.5766E-01	3.0659E-01	3.4478E-01	3.7095E-01	3.8467E-01	3.8467E-01
28	2.3851E-01	3.1669E-01	3.8331E-01	4.3562E-01	4.7163E-01	4.9045E-01	4.9045E-01
26	2.6945E-01	3.7293E-01	4.6188E-01	5.3221E-01	5.8882E-01	6.0617E-01	6.0617E-01
24	2.8471E-01	4.1811E-01	5.3384E-01	6.2598E-01	6.8999E-01	7.2333E-01	7.2333E-01
22	2.7810E-01	4.4460E-01	5.9042E-01	7.0744E-01	7.8927E-01	8.3194E-01	8.3194E-01
20	2.4965E-01	4.4946E-01	6.2626E-01	7.6943E-01	8.7039E-01	9.2339E-01	9.2339E-01
18	2.0914E-01	4.3832E-01	6.4333E-01	8.1093E-01	9.3035E-01	9.9386E-01	9.9386E-01
16	1.7164E-01	4.2317E-01	6.5039E-01	8.3777E-01	9.7267E-01	1.0458E+00	1.0458E+00
14	1.4722E-01	4.1393E-01	6.5656E-01	8.5794E-01	1.0042E+00	1.0851E+00	1.0851E+00
12	1.3680E-01	4.1342E-01	6.6605E-01	8.7654E-01	1.0303E+00	1.1172E+00	1.1172E+00
10	1.3596E-01	4.1928E-01	6.7846E-01	8.9479E-01	1.0534E+00	1.1448E+00	1.1448E+00
8	1.3974E-01	4.2976E-01	6.9170E-01	9.1197E-01	1.0738E+00	1.1687E+00	1.1687E+00
6	1.4485E-01	4.3688E-01	7.0405E-01	9.2719E-01	1.0914E+00	1.1890E+00	1.1890E+00
4	1.4971E-01	4.4483E-01	7.1474E-01	9.4014E-01	1.1060E+00	1.2060E+00	1.2060E+00
2	1.5397E-01	4.5178E-01	7.2405E-01	9.5137E-01	1.1187E+00	1.2206E+00	1.2206E+00
1	1.5397E-01	4.5178E-01	7.2405E-01	9.5137E-01	1.1187E+00	1.2206E+00	1.2502E+00

**** Converged solution at time level 360

Convergence history for continuity equation

level	chgmax	chgavg	**** Without artificial viscosity ****				**** With artificial viscosity ****			
			resl2	resavg	resmax	resmax loc.	resl2	resavg	resmax	resmax loc.
1	0.000E+00	0.000E+00	1.139E+01	1.733E-01	6.048E-01	(17, 22)	1.139E+01	1.734E-01	6.049E-01	(17, 22)
2	0.000E+00	0.000E+00	9.262E+01	7.788E-01	1.695E+01	(2, 2)	9.299E+01	7.842E-01	1.711E+01	(2, 2)
10	4.162E-03	0.000E+00	4.151E+01	4.636E-01						

130	3.016E-05	0.000E+00	5.796E+01	2.459E-01	1.494E+01	(2, 50)	1.034E+00	6.303E-03	2.848E-01	(12, 50)
140	1.889E-05	0.000E+00	5.794E+01	2.445E-01	1.523E+01	(2, 50)	7.084E-01	4.216E-03	1.948E-01	(12, 50)
150	1.106E-05	0.000E+00	5.791E+01	2.438E-01	1.542E+01	(2, 50)	4.632E-01	2.814E-03	1.299E-01	(12, 50)
160	6.477E-06	0.000E+00	5.782E+01	2.432E-01	1.555E+01	(2, 50)	2.855E-01	1.733E-03	7.868E-02	(12, 50)
170	4.252E-06	0.000E+00	5.774E+01	2.427E-01	1.562E+01	(2, 50)	1.678E-01	1.070E-03	4.510E-02	(12, 50)
180	2.799E-06	0.000E+00	5.767E+01	2.423E-01	1.565E+01	(2, 50)	9.543E-02	6.349E-04	2.308E-02	(12, 50)
190	1.775E-06	0.000E+00	5.761E+01	2.420E-01	1.566E+01	(2, 50)	5.380E-02	3.829E-04	1.071E-02	(2, 43)
200	1.131E-06	0.000E+00	5.756E+01	2.417E-01	1.567E+01	(2, 50)	3.227E-02	2.366E-04	6.114E-03	(2, 38)
210	6.984E-07	0.000E+00	5.753E+01	2.416E-01	1.566E+01	(2, 50)	2.062E-02	1.465E-04	4.296E-03	(2, 39)
220	4.313E-07	0.000E+00	5.751E+01	2.415E-01	1.566E+01	(2, 50)	1.380E-02	9.386E-05	3.282E-03	(13, 50)
230	2.902E-07	0.000E+00	5.750E+01	2.414E-01	1.566E+01	(2, 50)	9.118E-03	5.782E-05	2.337E-03	(13, 50)
240	1.912E-07	0.000E+00	5.749E+01	2.414E-01	1.566E+01	(2, 50)	5.812E-03	3.491E-05	1.453E-03	(13, 50)
250	1.236E-07	0.000E+00	5.749E+01	2.414E-01	1.565E+01	(2, 50)	3.580E-03	2.080E-05	8.041E-04	(2, 42)
260	8.020E-08	0.000E+00	5.749E+01	2.414E-01	1.565E+01	(2, 50)	2.168E-03	1.238E-05	4.735E-04	(2, 43)
270	5.164E-08	0.000E+00	5.749E+01	2.414E-01	1.565E+01	(2, 50)	1.342E-03	7.781E-06	2.708E-04	(2, 44)
280	3.312E-08	0.000E+00	5.749E+01	2.414E-01	1.565E+01	(2, 50)	8.733E-04	4.992E-06	1.731E-04	(2, 38)
290	2.074E-08	0.000E+00	5.749E+01	2.414E-01	1.565E+01	(2, 50)	5.895E-04	3.330E-06	1.268E-04	(2, 39)
300	1.250E-08	0.000E+00	5.749E+01	2.414E-01	1.565E+01	(2, 50)	3.975E-04	2.216E-06	8.019E-05	(2, 40)
310	7.394E-09	0.000E+00	5.749E+01	2.414E-01	1.565E+01	(2, 50)	2.596E-04	1.476E-06	5.800E-05	(2, 40)
320	5.014E-09	0.000E+00	5.749E+01	2.414E-01	1.565E+01	(2, 50)	1.621E-04	9.537E-07	3.748E-05	(2, 41)
330	3.334E-09	0.000E+00	5.749E+01	2.414E-01	1.565E+01	(2, 50)	9.735E-05	6.063E-07	2.245E-05	(2, 42)
340	2.212E-09	0.000E+00	5.749E+01	2.414E-01	1.565E+01	(2, 50)	5.801E-05	3.863E-07	1.262E-05	(2, 43)
350	1.465E-09	0.000E+00	5.749E+01	2.414E-01	1.565E+01	(2, 50)	3.627E-05	2.501E-07	6.887E-06	(2, 44)

Convergence history for x-momentum equation

level	chgmax	chgavg	**** Without artificial viscosity ****				**** With artificial viscosity ****			
			resl2	resavg	resmax	resmax loc.	resl2	resavg	resmax	resmax loc.
1	0.000E+00	0.000E+00	1.442E+04	3.490E+01	3.328E+03	(26, 2)	1.442E+04	3.494E+01	3.330E+03	(26, 2)
2	0.000E+00	0.000E+00	1.435E+03	1.669E+01	5.483E+02	(14, 2)	3.166E+03	1.670E+01	5.466E+02	(14, 2)
10	7.992E-02	0.000E+00	1.622E+02	2.281E+00	2.054E+01	(50, 50)	1.498E+02	2.203E+00	1.360E+01	(13, 50)
20	2.838E-02	0.000E+00	1.337E+02	1.661E+00	1.355E+01	(2, 48)	1.228E+02	1.553E+00	3.156E+01	(13, 50)
30	2.415E-02	0.000E+00	1.939E+02	1.730E+00	2.995E+01	(50, 50)	1.139E+02	1.241E+00	2.903E+01	(13, 50)
40	1.921E-02	0.000E+00	2.080E+02	1.639E+00	3.009E+01	(2, 48)	9.121E+01	8.762E-01	1.258E+01	(2, 34)
50	1.391E-02	0.000E+00	2.396E+02	1.705E+00	3.783E+01	(2, 40)	8.042E+01	7.733E-01	1.150E+01	(2, 35)
60	1.009E-02	0.000E+00	2.657E+02	1.690E+00	4.585E+01	(2, 40)	6.734E+01	5.756E-01	1.008E+01	(2, 36)
70	7.032E-03	0.000E+00	2.858E+02	1.725E+00	5.250E+01	(2, 40)	5.733E+01	4.977E-01	8.304E+00	(2, 37)
80	5.108E-03	0.000E+00	3.072E+02	1.751E+00	5.841E+01	(2, 40)	4.824E+01	3.892E-01	7.044E+00	(2, 38)
90	3.593E-03	0.000E+00	3.231E+02	1.777E+00	6.328E+01	(2, 41)	3.970E+01	3.097E-01	5.742E+00	(2, 38)
100	2.543E-03	0.000E+00	3.377E+02	1.810E+00	6.765E+01	(2, 41)	3.204E+01	2.378E-01	4.690E+00	(2, 39)
110	1.826E-03	0.000E+00	3.491E+02	1.838E+00	7.106E+01	(2, 41)	2.472E+01	1.747E-01	3.647E+00	(2, 40)
120	1.304E-03	0.000E+00	3.580E+02	1.867E+00	7.354E+01	(2, 41)	1.832E+01	1.244E-01	2.703E+00	(2, 40)
130	9.210E-04	0.000E+00	3.644E+02	1.893E+00	7.518E+01	(2, 41)	1.284E+01	8.358E-02	1.940E+00	(2, 41)
140	6.173E-04	0.000E+00	3.686E+02	1.914E+00	7.623E+01	(2, 42)	8.532E+00	5.434E-02	1.285E+00	(2, 42)
150	3.911E-04	0.000E+00	3.712E+02	1.929E+00	7.742E+01	(2, 42)	5.346E+00	3.390E-02	8.158E-01	(2, 42)
160	2.398E-04	0.000E+00	3.726E+02	1.939E+00	7.731E+01	(2, 42)	3.167E+00	2.036E-02	4.777E-01	(6, 37)
170	1.426E-04	0.000E+00	3.732E+02	1.944E+00	7.742E+01	(2, 42)	1.793E+00	1.192E-02	2.666E-01	(6, 37)
180	8.568E-05	0.000E+00	3.734E+02	1.946E+00	7.740E+01	(2, 42)	1.000E+00	6.897E-03	1.561E-01	(2, 37)
190	4.211E-05	0.000E+00	3.733E+02	1.947E+00	7.734E+01	(2, 42)	5.817E-01	4.053E-03	1.031E-01	(2, 38)
200	4.274E-05	0.000E+00	3.732E+02	1.946E+00	7.728E+01	(2, 42)	3.692E-01	2.549E-03	6.774E-02	(2, 39)
210	2.809E-05	0.000E+00	3.731E+02	1.946E+00	7.725E+01	(2, 42)	2.483E-01	1.636E-03	4.435E-02	(2, 40)
220	1.784E-05	0.000E+00	3.730E+02	1.946E+00	7.719E+01	(2, 42)	1.670E-01	1.066E-03	2.852E-02	(2, 41)
230	1.099E-05	0.000E+00	3.729E+02	1.945E+00	7.717E+01	(2, 42)	1.084E-01	6.776E-04	1.768E-02	(2, 42)
240	6.643E-06	0.000E+00	3.729E+02	1.945E+00	7.717E+01	(2, 42)	6.757E-02	4.188E-04	1.034E-02	(2, 43)
250	3.966E-06	0.000E+00	3.729E+02	1.945E+00	7.716E+01	(2, 42)	4.112E-02	2.596E-04	6.072E-03	(6, 38)
260	2.357E-06	0.000E+00	3.729E+02	1.945E+00	7.717E+01	(2, 42)	2.529E-02	1.642E-04	4.161E-03	(2, 37)
270	1.605E-06	0.000E+00	3.729E+02	1.945E+00	7.717E+01	(2, 42)	1.628E-02	1.052E-04	2.860E-03	(2, 38)
280	1.125E-06	0.000E+00	3.729E+02	1.945E+00	7.717E+01	(2, 42)	1.093E-02	6.838E-05	1.947E-03	(2, 39)
290	7.539E-07	0.000E+00	3.729E+02	1.945E+00	7.717E+01	(2, 42)	7.388E-03	4.437E-05	1.298E-03	(2, 40)
300	4.869E-07	0.000E+00	3.729E+02	1.945E+00	7.717E+01	(2, 42)	4.856E-03	2.812E-05	8.326E-04	(2, 41)
310	3.050E-07	0.000E+00	3.729E+02	1.945E+00	7.717E+01	(2, 42)	3.048E-03	1.772E-05	5.045E-04	(2, 42)
320	1.871E-07	0.000E+00	3.729E+02	1.945E+00	7.717E+01	(2, 42)	1.827E-03	1.124E-05	2.878E-04	(2, 42)
330	1.133E-07	0.000E+00	3.729E+02	1.945E+00	7.717E+01	(2, 42)	1.072E-03	7.047E-06	1.671E-04	(2, 36)
340	6.849E-08	0.000E+00	3.729E+02	1.945E+00	7.717E+01	(2, 42)	6.529E-04	4.528E-06	1.163E-04	(2, 37)
350	4.325E-08	0.000E+00	3.729E+02	1.945E+00	7.717E+01	(2, 42)	4.371E-04	3.043E-06	8.111E-05	(2, 38)

Convergence history for y-momentum equation

level	chgmax	chgavg	**** Without artificial viscosity ****				**** With artificial viscosity ****			
			resl2	resavg	resmax	resmax loc.	resl2	resavg	resmax	resmax loc.
1	0.000E+00	0.000E+00	8.313E+03	2.209E+01	1.660E+03	(14, 2)	8.317E+03	2.212E+01	1.661E+03	(14, 2)
2	0.000E+00	0.000E+00	1.932E+03	8.514E+00	4.013E+02	(10, 2)	1.934E+03	8.537E+00	4.015E+02	(10, 2)
10	5.181E-02	0.000E+00	1.344E+02	1.665E+00	2.139E+01	(33, 50)	1.118E+02	1.609E+00	7.824E+00	(34, 49)
20	2.366E-02	0.000E+00	9.622E+01	1.311E+00	1.056E+01	(15, 50)	8.934E+01	1.227E+00	1.293E+01	(12, 50)
30	1.750E-02	0.000E+00	1.733E+02	1.449E+00	3.263E+01	(31, 50)	8.236E+01	9.646E-01	9.482E+00	(13, 50)
40	1.089E-02	0.000E+00	1.479E+02	1.287E+00	2.512E+01	(34, 50)	7.754E+01	7.628E-01	9.728E+00	(6, 27)
50	7.577E-03	0.000E+00	1.661E+02	1.307E+00	3.016E+01	(34, 50)	7.542E+01	6.961E-01	1.062E+01	(5, 29)
60	5.192E-03	0.000E+00	1.690E+02	1.211E+00	3.088E+01	(35, 50)	6.774E+01	5.359E-01	1.077E+01	(5, 30)
70	3.734E-03	0.000E+00	1.650E+02	1.157E+00	3.015E+01	(35, 50)	6.015E+01	4.650E-01	1.043E+01	(5, 31)
80	2.723E-03	0.000E+00	1.685E+02	1.114E+00	3.099E+01	(35, 50)	5.312E+01	3.777E-01	9.873E+00	(5, 32)
90	1.988E-03	0.000E+00	1.665E+02	1.074E+00	3.016E+01	(36, 50)	4.603E+01	3.144E-01	9.073E+00	(5, 33)
100	1.400E-03	0.000E+00	1.675E+02	1.045E+00	3.014E+01	(36, 50)	3.879E+01	2.507E-01	7.924E+00	(5, 34)
110	9.740E-04	0.000E+00	1.675E+02	1.019E+00	2.969E+01	(36, 50)	3.129E+01	1.930E-01	6.478E+00	(5, 35)
120	6.436E-04	0.000E+00	1.678E+02	1.000E+00	2.939E+01	(36, 50)	2.409E+01	1.426E-01	5.181E+00	(4, 36)
130	4.109E-04	0.000E+00	1.683E+02	9.901E-01	2.909E+01	(36, 50)	1.760E+01	9.995E-02	3.862E+00	(4, 37)
140	2.843E-04	0.000E+00	1.686E+02	9.883E-01	2.881E+01	(36, 50)	1.219E+01	6.738E-02	2.781E+00	(4, 37)
150	1.901E-04	0.000E+00	1.689E+02	9.911E-01	2.860E+01	(36, 50)	8.011E+00	4.305E-02	1.863E+00	(4, 38)
160	1.260E-04	0.000E+00	1.691E+02	9.935E-01	2.841E+01	(36, 50)	5.022E+00	2.671E-02	1.151E+00	(4, 39)
170	8.078E-05	0.000E+00	1.692E+02	9.942E-01	2.828E+01	(36, 50)	3.040E+00	1.637E-02	6.930E-01	(4, 39)
180	5.018E-05	0.000E+00	1.692E+02	9.938E-01	2.819E+01	(36, 50)	1.812E+00	9.946E-03	3.958E-01	(4, 40)
190	3.047E-05	0.000E+00	1.691E+02	9.931E-01	2.813E+01	(36, 50)	1.091E+00	6.066E-03	2.365E-01	(4, 34)
200	1.853E-05	0.000E+00	1.691E+02	9.921E-01	2.809E+01	(36, 50)	6.785E-01	3.707E-03	1.587E-01	(4, 35)
210	1.184E-05	0.000E+00	1.690E+02	9.911E-01	2.806E+01	(36, 50)	4.342E-01	2.325E-03	1.048E-01	(4, 36)
220	7.755E-06	0.000E+00	1.690E+02	9.904E-01	2.805E+01	(36, 50)	2.806E-01	1.293E-03	6.807E-02	(4, 37)
230	5.010E-06	0.000E+00	1.690E+02	9.899E-01	2.804E+01	(36, 50)	1.796E-01	9.463E-04	4.309E-02	(4, 38)
240	3.236E-06	0.000E+00	1.689E+02	9.894E-01	2.804E+01	(36, 50)	1.132E-01	5.904E-04	2.668E-02	(4, 38)
250	2.078E-06	0.000E+00	1.689E+02	9.895E-01	2.804E+01	(36, 50)	7.785E-02	3.675E-04	1.629E-02	(4, 39)
260	1.324E-06	0.000E+00	1.689E+02	9.894E-01	2.804E+01	(36, 50)	4.453E-02	2.330E-04	9.662E-03	(4, 40)
270	8.324E-07	0.000E+00	1.689E+02	9.894E-01	2.804E+01	(36, 50)	2.867E-02	1.500E-04	6.004E-03	(4, 34)
280	5.134E-07	0.000E+00	1.689E+02	9.895E-01	2.804E+01	(36, 50)	1.885E-02	9.854E-05	4.243E-03	(4, 35)
290	3.148E-07	0.000E+00	1.689E+02	9.895E-01	2.804E+01	(36, 50)	1.246E-02	6.511E-05	2.916E-03	(4, 36)
300	2.079E-07	0.000E+00	1.689E+02	9.895E-01	2.804E+01	(36, 50)	8.120E-03	4.225E-05	1.929E-03	(4, 37)
310	1.358E-07	0.000E+00	1.689E+02	9.895E-01	2.804E+01	(36, 50)	5.156E-03	2.669E-05	1.218E-03	(4, 38)
320	8.785E-08	0.000E+00	1.689E+02	9.895E-01	2.804E+01	(36, 50)	3.188E-03	1.655E-05	7.634E-04	(4, 38)

```

330 5.756E-08 0.000E+00 1.689E+02 9.895E-01 2.804E+01 ( 36, 50) 1.946E-03 1.035E-05 4.567E-04 ( 4, 39)
340 3.730E-08 0.000E+00 1.689E+02 9.895E-01 2.804E+01 ( 36, 50) 1.204E-03 6.586E-06 2.643E-04 ( 4, 40)
350 2.386E-08 0.000E+00 1.689E+02 9.895E-01 2.804E+01 ( 36, 50) 7.772E-04 4.301E-06 1.651E-04 ( 4, 34)
STOP (called by MAIN )

```

This case used 18.2 seconds of CPU time.

Computed Results

In Figure 9.4 the computed static pressure coefficient, defined as $(\bar{p} - p_\infty)/(\rho U_\infty^2/2)$, is plotted as a function of θ for both the Euler and viscous flow cases. Also shown are the experimental data of Grove, Shair, Petersen, and Acrivos (1964), and the exact solution for potential flow.

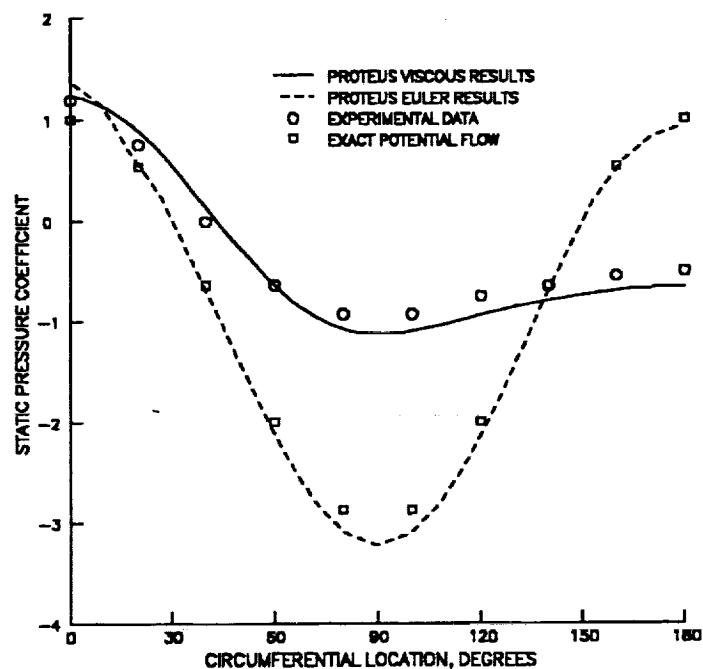


Figure 9.4 - Pressure coefficient for flow past a circular cylinder.

9.3 TRANSONIC DIFFUSER FLOW

Problem Description

In this test case, transonic turbulent flow was computed in a converging-diverging duct. The flow entered the duct subsonically, accelerated through the throat to supersonic speed, then decelerated through a normal shock and exited the duct subsonically. Extensive experimental data are available for flow through this duct (Chen, Sajben, and Kroutil, 1979; Bogar, Sajben and Kroutil, 1983; Salmon, Bogar, and Sajben, 1983; Sajben, Bogar, and Kroutil, 1984; Bogar, 1986). The computational domain is shown in Figure 9.5.

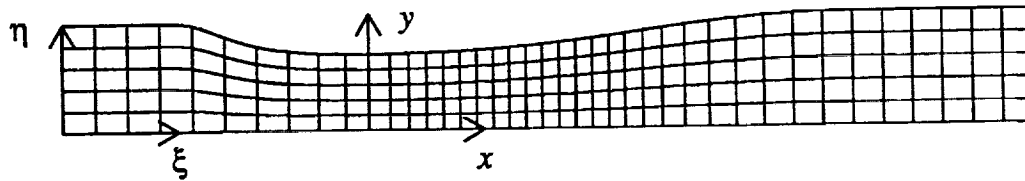


Figure 9.5 - Computational domain for transonic diffuser flow.

Reference Conditions

The throat height of 0.14435 ft. was used as the reference length L_r . The reference velocity u_r was 100 ft/sec. The reference temperature and density were 525.602 °R and 0.1005 lb_m/ft³, respectively. These values match the inlet total temperature and total pressure used in other numerical simulations of this flow (Hsieh, Bogar, and Coakley, 1987).

Computational Coordinates

The x coordinate for this duct runs from -4.04 to $+8.65$. The Cartesian coordinates of the bottom wall are simply $y = 0$ for all x . For the top wall, the y coordinate is given by (Bogar, Sajben, and Kroutil, 1983)

$$y = \begin{cases} 1.4144 & \text{for } x \leq -2.598 \\ \alpha \cosh \zeta / (\alpha - 1 + \cosh \zeta) & \text{for } -2.598 < x < 7.216 \\ 1.5 & \text{for } x \geq 7.216 \end{cases}$$

where the parameter ζ is defined as

$$\zeta = \frac{C_1(x/x_l)[1 + C_2x/x_l]^{C_3}}{(1 - x/x_l)^{C_4}}$$

The various constants used in the formula for the top wall height in the converging ($-2.598 \leq x \leq 0$) and diverging ($0 \leq x \leq 7.216$) parts of the duct are given in the following table.

Constant	Converging	Diverging
α	1.4114	1.5
x_l	-2.598	7.216
C_1	0.81	2.25
C_2	1.0	0.0
C_3	0.5	0.0
C_4	0.6	0.6

A body-fitted coordinate system was generated for this duct and stored in an unformatted file to be read by *Proteus*. The Cray runstream used to generate this file is listed below. Note that the Fortran program is included in the JCL as a UNICOS "here document".

```
# QSUB -lM 1.0mW -lT 60
# QSUB -mb -me
# QSUB -o temp.out
# QSUB -eo
# QSUB -r sajben0
# QSUB -s /bin/sh
set -x
ja
touch sajben.coords
ln sajben.coords fort.7
cat > temp.f << EOF
c
c          This program generates a coordinate system for the 2-d
c          converging-diverging Sajben nozzle.
c
c-----Fortran variable definitions
c
c          nx      - number of grid points in x direction
c          ny      - number of grid points in y direction
c          nit      - number of smoothing iterations
c          nsm      - spatial extent of smoothing region
c          np2      - x index at xp2
c          np1      - x index at xpl
c          xmin     - minimum value of x
c          xmax     - maximum value of x
c          xpl      - start of fine grid
c          xp2      - end of fine grid
c
c-----Output computed:
c
c          x(i,j) - x coordinates at grid points
c          y(i,j) - y coordinates at grid points
c
c          dimension p(150),q(150)
c          dimension x(150,150),y(150,150)
c
c-----Set constants for coordinates of top wall. "m" for converging
c-----          section, "p" for diverging section.
c
c          data alfam,d1m,c1m,c2m,c3m,c4m /1.4114,-2.598,0.81,1.0,0.5,0.6/
c          data alfap,d1p,clp,c2p,c3p,c4p /1.5      , 7.216,2.25,0.0,0.0,0.6/
c
c-----Define parameters
c
c          xmin = -4.04
c          xmax = 8.650
c          xpl = 0.0
c          xp2 = 4.0
c          nx = 81
c          ny = 51
c          ymin = 0.0
c          ymax = 1.0
c          np1 = 20
c          np2 = 57
c          xinc1 = (xpl - xmin)/(np1 - 1)
c          xinc2 = (xp2 - xpl)/(np2 - np1)
c          xinc3 = (xmax - xp2)/(nx - np2)
c          yincr = (ymax - ymin)/(ny - 1)
c          nit = 8
c
c-----Set up fine and coarse grid
c
c          p(1) = xmin
c          do 10 i = 2,nx
c            if (i .le. np1) xincr = xinc1
c            if (i .gt. np1 .and. i .le. np2) xincr = xinc2
```

```

        if (i .gt. np2) xincr = xinc3
        p(i) = p(i-1) + xincr
10      continue
c
c-----Smooth mesh by averaging between np1 and np2
c
        do 30 iter = 1,nit
        do 20 i = 2,nx-1
        p(i) = (p(i-1) + p(i+1))*0.5
20      continue
30      continue
c
c-----Generate coordinates of top wall
c
        do 100 i = 1,nx
        if (p(i) .lt. 0.0) then
c-----Converging section
            xbar = p(i)/dlm
            xd = 1. - xbar
            if (xd .le. 0.0) xd = 1.e-3
            xi = clm*xbar*((1.+c2m*xbar)**c3m)/(xd**c4m)
            q(i) = alfam*cosh(xi)/((alfam-1.0) + cosh(xi))
        else if (p(i) .ge. 0.0) then
c-----Diverging section
            xbar = p(i)/dlp
            xd = 1. - xbar
            if (xd .le. 0.0) xd = 1.e-3
            xi = clp*xbar*((1.+c2p*xbar)**c3p)/(xd**c4p)
            q(i) = alfap*cosh(xi)/((alfap-1.0) + cosh(xi))
        endif
100     continue
c
c-----Generate x and y arrays
c
        do 200 i = 1,nx
        do 200 j = 1,ny
        x(i,j) = p(i)
        ytemp = (j-1)*yincr + ymin
        y(i,j) = q(i)*ytemp/ymax
200     continue
c
c-----Write unformatted coordinate system file
c
        write (7) nx,ny
        write (7) ((x(i,j),i=1,nx),j=1,ny),
        $          ((y(i,j),i=1,nx),j=1,ny)
        stop
        end
EOF
cft77 -d pq temp.f
segldr -o temp.ex temp.o
temp.ex
ja -cslt

```

The resulting body-fitted coordinate system is shown in Figure 9.5. For clarity, the grid points are thinned by factors of 2 and 10 in the x and y directions, respectively. Note that for good resolution of the flow near the normal shock, the grid defining the computational coordinate system is denser in the x direction in the region just downstream of the throat. In the y direction, the *computational coordinates* are evenly spaced. The *computational mesh*, however, was tightly packed near both walls to resolve the turbulent boundary layers.³⁴

³⁴ The distinction between the computational coordinate system and the computational mesh is explained in Section 2.2.

Initial Conditions

The initial conditions were simply zero velocity and constant pressure and temperature. Thus, $u = v = 0$ and $p = T = 1$ everywhere in the flow field.

Boundary Conditions

This calculation was performed in three separate runs. In the first run, the exit static pressure was gradually lowered to a value low enough to establish supersonic flow throughout the diverging portion of the duct. The pressure was lowered as follows:

$$p(t) = \begin{cases} 0.99 & \text{for } 1 < n \leq 100 \\ -2.1405 \times 10^{-3}n + 1.20405 & \text{for } 101 \leq n \leq 500 \\ 0.1338 & \text{for } 501 \leq n \leq 3001 \end{cases}$$

where n is the time level. The equation for p for $101 \leq n \leq 500$ is simply a linear interpolation between $p = 0.99$ and $p = 0.1338$. In the second run, the exit pressure was gradually raised to a value consistent with the formation of a normal shock just downstream of the throat. Thus,

$$p(t) = \begin{cases} 3.4327 \times 10^{-4}n - 0.89636 & \text{for } 3001 < n \leq 5000 \\ 0.82 & \text{for } 5001 \leq n \leq 6001 \end{cases}$$

Again, the equation for p for $3001 < n \leq 5000$ is simply a linear interpolation between $p = 0.1338$ and $p = 0.82$. In the third run, the exit pressure was kept constant at 0.82.

The remaining boundary conditions were the same for all runs. At the inlet, constant total pressure and temperature were specified, and the y -velocity and the normal gradient of the x -velocity were both set equal to zero. At the exit, the normal gradients of temperature and both velocity components were set equal to zero. At both walls, no-slip adiabatic conditions were used, and the normal pressure gradient was set equal to zero.

These boundary conditions are summarized in the following table.

Boundary	Boundary Conditions
$\xi = 0$	$\partial u / \partial n = 0, v = 0, p_T = T_T = 1$
$\xi = 1$	$\partial u / \partial n = \partial v / \partial n = 0, p = p(t), \partial T / \partial n = 0$
$\eta = 0$	$u = v = 0, \partial p / \partial n = \partial T / \partial n = 0$
$\eta = 1$	$u = v = 0, \partial p / \partial n = \partial T / \partial n = 0$

Proteus JCL and Input File

The Cray UNICOS job control language used for the first run is listed below, including the namelist input. In namelist BC, note that at $\xi = 1$ the fourth boundary condition type is specified by JBC1(4,2) as $p = f$, and that the value f is specified by the table of GTBC1(,4,2) vs. NTBCA. In namelist NUM, the values of CAVS2E and CAVS4E are based on experience for internal flows with normal shocks. The optimum values for these parameters seem to be a function of the type of flow, the type of time differencing, and the size of the time step. In namelist TIME, a small CFL value is being used. This minimized starting transients and enhanced stability when using an unsteady pressure boundary condition. Note that the time level will run from 1 to 3001, but that the time levels in the unsteady boundary condition table only run from 100 to 500. This causes a warning message to be printed. For $n < 100$ the boundary condition value will be set equal to the first value in the table, and for $n > 500$ it will be set equal to the last value in the table.

The input includes a brief explanation of each line, and should be compared with the detailed input description in Section 3.0. This JCL is essentially the same as the example presented in Section 8.3, but with lines eliminated that are not applicable to this case.

```
# QSUB -lM 2.0mW -lT 1800
# QSUB -mb -me
# QSUB -o temp.out
# QSUB -eo
# QSUB -r sajben1
# QSUB -s /bin/sh
set -x
ja

# Create modifications file

cat > mods << EOM
*id temp
*d params1.19
    parameter (nlp = 81, n2p = 51)
EOM

# Set up and link necessary input/output files

touch plotx
touch plotq
touch chist
touch rql
touch rxl
ln sajben.coords fort.7
ln plotx fort.8
ln plotq fort.9
ln chist fort.10
ln rql fort.12
ln rxl fort.14

# Update code using modifications file

update -p p2d20.u -i mods -c temp -f
cft77 -d pq -o aggress temp.f

# Load and run

sejldr -o temp.ex temp.o
temp.ex << EOD
Transonic Diffuser Flow, Run 1
&rstrt
    irest=1,
&end
&io
    ivout=1,4,30,47*0,
    iwout2=-1,-1,
    iprt=10000,
    iprt1=2, iprt2=1,
    iplot=3,
&end
&gmtry
    ngeom=10,
&end
&flow
    lr=0.14435,
    ur=100.,
    tr=525.602,
    rhor=0.1005,
    gamr=1.4,
&end
&bc
    jbc1(1,1)=57, jbc1(1,2)=53, gbc1(1,1)=1., gbc1(1,2)=0.,
    TT=1, ∂T/∂n=0
    at ξ = 0, 1.
```

Write restart files.

Print u, M, p.

Print parameters along η boundaries.

Print every 10,000 time levels.

Print at every other ξ index, every η index.

Write PLOT3D 3-D plot files.

Get computational coordinates from file.

Set L_r.

Set u_r.

Set T_r.

Set ρ_r.

Constant specific heats.

```

jbc1(2,1)=13, jbc1(2,2)=13, gbc1(2,1)=0., gbc1(2,2)=0.,  $\partial u/\partial n = 0$  at  $\xi = 0, 1.$ 
jbc1(3,1)=21, jbc1(3,2)=23, gbc1(3,1)=0., gbc1(3,2)=0.,  $v = 0, \partial v/\partial n = 0$ 
                                                    at  $\xi = 0, 1.$ 
jbc1(4,1)=47, jbc1(4,2)=41, gbc1(4,1)=1.,  $p_T = 1, p$  specified
                                                    at  $\xi = 0, 1.$ 
jbc2(1,1)=53, jbc2(1,2)=53, gbc2(1,1)=0., gbc2(1,2)=0.,  $\partial T/\partial n = 0$  at  $\eta = 0, 1.$ 
jbc2(2,1)=11, jbc2(2,2)=11, gbc2(2,1)=0., gbc2(2,2)=0.,  $u = 0$  at  $\eta = 0, 1.$ 
jbc2(3,1)=21, jbc2(3,2)=21, gbc2(3,1)=0., gbc2(3,2)=0.,  $v = 0$  at  $\eta = 0, 1.$ 
jbc2(4,1)=43, jbc2(4,2)=43, gbc2(4,1)=0., gbc2(4,2)=0.,  $\partial p/\partial n = 0$  at  $\eta = 0, 1.$ 
jtbcl(4,2)=1, 4th. b. c. at  $\xi = 1$  is general unsteady.
ntbc=2, 2 values in b. c. table.
ntbca=100,500, Time levels in b. c. table.
gtbcl(1,4,2)=0.99, gtbcl(2,4,2)=0.1338, Values in b. c. table.
&end
&num
n1=81, n2=51, Use an  $81 \times 51$  mesh.
ipack(2)=1, Pack in  $\eta$  direction.
sq(2,1)=0.5, sq(2,2)=1.002, Pack fairly tightly near both  $\eta$  boundaries.
iav4e=2, iav2e=2, iav2i=0, Use nonlinear coefficient artificial viscosity.
cavs4e=4*0.005, cavs2e=4*0.1, cavs2i=4*0., Artificial viscosity coefficients.
&end
&time
idtmod=1, Recompute  $\Delta\tau$  every time step.
idtau=5, cfl=0.5, Spatially varying  $\Delta\tau$ .
ntime=3000, Limit of 3000 time steps.
ictest=1, eps=4*0.000001, Use  $\Delta Q_{max} = 10^{-6}$  as convergence criteria.
&end
&turb
iturb=1, Turbulent flow, Baldwin-Lomax model.
&end
&ic
Use default initial conditions.
&end
EOD
ja -cslt

```

The JCL and input for the second run were similar to that used for the first run. The only differences were:

1. The *touch* and *ln* commands for the restart files were changed to:

```

touch rq2
touch rx2
ln sajben.rq1 fort.11
ln sajben.rq2 fort.12
ln sajben.rx1 fort.13
ln sajben.rx2 fort.14

```

2. In the TITLE, Run 1 was changed to Run 2.
3. In namelist RSTRT, IREST was set equal to 2, to read and write restart files.
4. In namelist BC, the time levels NTBKA in the boundary condition table were 3001 and 5000, and the values GTBCL(1,4,2) and GTBCL(2,4,2) were 0.1338 and 0.82.

The JCL and input for the third run were similar to that used for the second run. The only differences were:

1. The *touch* and *ln* commands for the restart files were changed to:

```

touch rq3
touch rx3
ln sajben.rq2 fort.11
ln sajben.rq3 fort.12
ln sajben.rx2 fort.13
ln sajben.rx3 fort.14

```

2. In the TITLE, Run 2 was changed to Run 3.
3. In namelist BC, GBC1(4,2) was specified as 0.82. The parameters JTBC1(4,2), NTBC, NTBCA, and GTBC1 were defaulted.
4. In namelist NUM, CAVS4E was lowered to 5*0.0004.
5. In namelist TIME, CFL was raised to 5.0.

Note that restart files were written and saved at the end of the third run in case a fourth run was necessary. A fourth run was not used for this case, however.

Standard Proteus Output

The output listing for the third run is shown below. In the flow field printout, only the Mach number at the last time level is included.

NASA Lewis Research Center
Internal Fluid Mechanics Division
2-d Proteus version 2.0
January 1992

Transonic Diffuser Flow, Run 3

```

&RSTRT IREST = 2, NROIN = 11, NROOUT = 12, NRXIN = 13, NRXOUT = 14, &END
&IO IDEBUG = 20*0, IPLOT = 3, IPLT = 0, IPLTA = 101*0, IPRT = 10000, IPRT1 = 2, IPRT2 = 1, IPRTA = 101*0, IPRTIA = 1,
2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48, 50, 52,
54, 56, 58, 60, 62, 64, 66, 68, 70, 72, 74, 76, 78, 80, 81, 39*0, IPRT2A = 1, 2, 3, 4, 5, 6, 7, 8, 9,
10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35,
36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, IUNITS = 0, IVOUT = 1, 4, 30, 47*0,
IWOUT1 = 2*0, IWOUT2 = 2*-1, NGRID = 7, NHIST = 10, NHMAX = 100, NOUT = 6, NPLT = 9, NPLTX = 8, NSCR1 = 20, &END
&GMTRY IAXI = 0, NGEOM = 10, RMAX = 1.0, RMIN = 0.0, THMAX = 90.0, THMIN = 0.0, XMAX = 1.0, XMIN = 0.0, YMAX = 1.0,
YMIN = 0.0, &END
&FLOW GAMR = 1.4, HSTAGR = 0.0, ICVARS = 2, IEULER = 0, IHSTAG = 0, ILAMV = 0, ISWIRL = 0, ITHIN = 2*0, KTR = 0.0,
LR = 0.14435, MACHR = 0.0, MUR = 0.0, PRLR = 0.0, RER = 0.0, RG = 1716.0, RHOR = 0.1005, TR = 525.602, UR = 100.0, &END
&BC FBC1 = 408*0.0, FBC2 = 648*0.0, FBCT1 = 204*0.0, FBCT2 = 324*0.0, GBC1 = 1.0, 2*0.0, 1.0, 3*0.0, 0.82,
GBC2 = 8*0.0, GBCT1 = 4*0.0, GBCT2 = 4*0.0, GTBC1 = 80*0.0, GTBC2 = 80*0.0, IBC1 = 408*0.0, IBC2 = 648*0.0, IBCT1 = 51*0,
51*10, 51*0, 51*10, IBCT2 = 81*0, 81*10, 81*0, 81*10, JBC1 = 57, 13, 21, 47, 53, 13, 23, 41, JBC2 = 53, 11, 21,
43, 53, 11, 21, 43, JBCT1 = 0, 10, 0, 10, JBCT2 = 0, 10, 8, 10, JTBC1 = 8*0, JTBC2 = 8*0, KBC1 = 2*0,
KBC2 = 2*0, NTBC = 0, NTBCA = 10*0, &END
&NUM CAVS2E = 4*0.1, CAVS4E = 4*0.0, CAVS4E = 4*4.0E-04, IAV2E = 2, IAV2I = 0, IAV4E = 2, IPACK = 0, 1, N1 = 81,
N2 = 51, S0 = 0.0, 0.5, 10000.0, 1.002, THC = 1.0, 2*0.0, THE = 1.0, 2*0.0, THKE = 1.0, 0.0, THX = 1.0, 2*0.0,
THY = 1.0, 2*0.0, THZ = 1.0, 2*0.0, &END
&TIME CFL = 5.0, 9*1.0, CFLMAX = 10.0, CFLMIN = 0.5, CHG1 = 4.0E-02, CHG2 = 6.0E-02, DT = 10*1.0E-02, DTF1 = 1.25,
DTF2 = 1.25, DTHMAX = 1.0E-02, DTHMIN = 1.0E-02, EPS = 4*1.0E-06, ICHECK = 10, ICTEST = 1, IDTAU = 5, IDTMOD = 1,
NDTCYC = 2, NITAVG = 10, NTIME = 3000, 9*0, NTSEQ = 1, TLIM = 20.0, &END
&TURB APLUS = 26.0, CB = 5.5, CCLAU = 1.68E-02, CCP = 1.6, CKLEB = 0.3, CKMIN = 0.0, CMUR = 9.0E-02, CNA = 2.0,
CMV = 1.7, CONE = 1.35, CTWOR = 1.8, CTHREE = 1.15E-02, CVK = 0.4, CHK = 0.25, ILDAMP = 0, INNFR = 1, ITETA = 0,
ITURB = 1, ITXI = 1, IWALL1 = 2*-1, IWALL2 = 2*-1, LWALL1 = 102*0, LWALL2 = 162*0, NTKE = 1, PKT = 0.91, REXT1 = 0.0,
REXT2 = 0.0, TFACT = 1.0, &END

```

Normalizing conditions

```

Length, lr          = 1.4435E-01 ft
Velocity, ur         = 1.0000E+02 ft/sec
Temperature, tr      = 5.2560E+02 deg R
Density, rho         = 1.0050E-01 lbm/ft3
Viscosity, mu       = 1.2152E-05 lbm/ft-sec
Thermal conductivity, ktr = 1.0309E-01 lbm/ft-sec3-deg R
Pressure, rho*ur**2  = 1.0050E+03 lbm/ft-sec2
Energy/vol., rho*ur**2 = 1.0050E+03 lbm/ft-sec2
Gas constant, ur**2/tr = 1.9026E+01 ft2/sec2-deg R
Specific heat, ur**2/tr = 1.9026E+01 ft2/sec2-deg R
Stagnation enthalpy, ur**2 = 1.0000E+04 ft2/sec2
Time, lr/ur         = 1.4435E-03 sec

```

Reference conditions

```

Reynolds number, rer = 1.1938E+05
Mach number, machr   = 8.8992E-02
Specific heat ratio, gamr = 1.4000E+00
Laminar Prandtl number, prlr = 7.0801E-01
"Reference" Prandtl number, prr = 2.2428E-03
Specific heat at const. press. = 6.0060E+03 ft2/sec2-deg R
Specific heat at const. vol. = 4.2900E+03 ft2/sec2-deg R
Gas constant, rg     = 1.7160E+03 ft2/sec2-deg R
Pressure, pr         = 2.8173E+03 lbf/ft2
Stagnation enthalpy, hstagr = 3.1618E+06 ft2/sec2

```

Boundary condition parameters

```

jbc2(1,2) = 53   gbc2(1,2) = 0.0000E+00
jbc2(2,2) = 11   gbc2(2,2) = 0.0000E+00
jbc2(3,2) = 21   gbc2(3,2) = 0.0000E+00
jbc2(4,2) = 43   gbc2(4,2) = 0.0000E+00

jbc1(1,1) = 57   gbc1(1,1) = 1.0000E+00
jbc1(2,1) = 13   gbc1(2,1) = 0.0000E+00
jbc1(3,1) = 21   gbc1(3,1) = 0.0000E+00
jbc1(4,1) = 47   gbc1(4,1) = 1.0000E+00

jbc1(1,2) = 53   gbc1(1,2) = 0.0000E+00
jbc1(2,2) = 13   gbc1(2,2) = 0.0000E+00
jbc1(3,2) = 23   gbc1(3,2) = 0.0000E+00
jbc1(4,2) = 41   gbc1(4,2) = 8.2000E-01

```

```

jbc2(1,1) = 53   gbc2(1,1) = 0.0000E+00
jbc2(2,1) = 11   gbc2(2,1) = 0.0000E+00
jbc2(3,1) = 21   gbc2(3,1) = 0.0000E+00
jbc2(4,1) = 43   gbc2(4,1) = 0.0000E+00

```

Mach number at time level 9001

ixi =	1	2	4	6	8	10	12	14	16	18
jbc2	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
51	1.8496E-01	1.8757E-01	1.8201E-01	1.5459E-01	1.2994E-01	1.8357E-01	2.3634E-01	2.8125E-01	2.9193E-01	2.9279E-01
50	2.5903E-01	2.6202E-01	2.5812E-01	2.2602E-01	1.9081E-01	2.5791E-01	3.2673E-01	4.0066E-01	4.2434E-01	4.3090E-01
49	3.0048E-01	3.0339E-01	2.9784E-01	2.6447E-01	2.2135E-01	2.9508E-01	3.7407E-01	4.6242E-01	4.9105E-01	5.0205E-01
48	3.3486E-01	3.3751E-01	3.2632E-01	2.9240E-01	2.4188E-01	3.2062E-01	4.1336E-01	5.1184E-01	5.4041E-01	5.5084E-01
47	3.6652E-01	3.6875E-01	3.4981E-01	3.1656E-01	2.5866E-01	3.4079E-01	4.5318E-01	5.6220E-01	5.8869E-01	5.9363E-01
46	3.9512E-01	3.9682E-01	3.7047E-01	3.3849E-01	2.7407E-01	3.5770E-01	4.9366E-01	6.1330E-01	6.4170E-01	6.3952E-01
45	4.1973E-01	4.2085E-01	3.8902E-01	3.5912E-01	2.8927E-01	3.7216E-01	5.3142E-01	6.5975E-01	6.9808E-01	6.9231E-01
44	4.3944E-01	4.4000E-01	4.0563E-01	3.7855E-01	3.0477E-01	3.8670E-01	5.6384E-01	6.9757E-01	7.5151E-01	7.5049E-01
43	4.5059E-01	4.5077E-01	4.2020E-01	3.9651E-01	3.2073E-01	3.9594E-01	5.8956E-01	7.2555E-01	7.9532E-01	8.0813E-01
42	4.5230E-01	4.5239E-01	4.3231E-01	4.1240E-01	3.3707E-01	4.0658E-01	6.0648E-01	7.4477E-01	8.2604E-01	8.5769E-01
41	4.5144E-01	4.5156E-01	4.4158E-01	4.2539E-01	3.5344E-01	4.1692E-01	6.1624E-01	7.5724E-01	8.4399E-01	8.9494E-01
39	4.5249E-01	4.5259E-01	4.4809E-01	4.3459E-01	3.6926E-01	4.2707E-01	6.2321E-01	7.6503E-01	8.5453E-01	9.0985E-01
38	4.5481E-01	4.5486E-01	4.5234E-01	4.3951E-01	3.8370E-01	4.3705E-01	6.2941E-01	7.6979E-01	8.6038E-01	9.1679E-01
37	4.5636E-01	4.5637E-01	4.5436E-01	4.4118E-01	3.9580E-01	4.4655E-01	6.3429E-01	7.7262E-01	8.6217E-01	9.1901E-01
36	4.5657E-01	4.5657E-01	4.5473E-01	4.4263E-01	4.0477E-01	4.5490E-01	6.3733E-01	7.7425E-01	8.6431E-01	9.2200E-01
35	4.5689E-01	4.5688E-01	4.5528E-01	4.4449E-01	4.1113E-01	4.6177E-01	6.3835E-01	7.7488E-01	8.6775E-01	9.2659E-01
34	4.5761E-01	4.5758E-01	4.5615E-01	4.4620E-01	4.1661E-01	4.6732E-01	6.3726E-01	7.7436E-01	8.6812E-01	9.2826E-01
33	4.5748E-01	4.5744E-01	4.5597E-01	4.4651E-01	4.2075E-01	4.7202E-01	6.3589E-01	7.7207E-01	8.6597E-01	9.2768E-01
32	4.5842E-01	4.5839E-01	4.5684E-01	4.4606E-01	4.2544E-01	4.7754E-01	6.3053E-01	7.6691E-01	8.6360E-01	9.2702E-01
31	4.5813E-01	4.5810E-01	4.5648E-01	4.4544E-01	4.3300E-01	4.8588E-01	6.2895E-01	7.6023E-01	8.5846E-01	9.2644E-01
30	4.5882E-01	4.5876E-01	4.5707E-01	4.5019E-01	4.4200E-01	4.9529E-01	6.2774E-01	7.5353E-01	8.5212E-01	9.1969E-01
29	4.5742E-01	4.5732E-01	4.5570E-01	4.5321E-01	4.5085E-01	5.0418E-01	6.2615E-01	7.4646E-01	8.4524E-01	9.1488E-01
28	4.5817E-01	4.5805E-01	4.5610E-01	4.5671E-01	4.5855E-01	5.1277E-01	6.2494E-01	7.3983E-01	8.3923E-01	9.1043E-01
27	4.5894E-01	4.5881E-01	4.5640E-01	4.6024E-01	4.6802E-01	5.2020E-01	6.2343E-01	7.3613E-01	8.3370E-01	9.0712E-01
26	4.5966E-01	4.5953E-01	4.6080E-01	4.6378E-01	4.7539E-01	5.2659E-01	6.2290E-01	7.2915E-01	8.2835E-01	9.0325E-01
25	4.6081E-01	4.6012E-01	4.6197E-01	4.6652E-01	4.8134E-01	5.3156E-01	6.2186E-01	7.2403E-01	8.2322E-01	8.9925E-01
24	4.6081E-01	4.6072E-01	4.6313E-01	4.6888E-01	4.8581E-01	5.3523E-01	6.2097E-01	7.2128E-01	8.1921E-01	8.9588E-01
23	4.6145E-01	4.6138E-01	4.6433E-01	4.7112E-01	4.8969E-01	5.3851E-01	6.2054E-01	7.1863E-01	8.1660E-01	8.9415E-01
22	4.6190E-01	4.6185E-01	4.6523E-01	4.7274E-01	4.9259E-01	5.4088E-01	6.2017E-01	7.1686E-01	8.1454E-01	8.9263E-01
21	4.6234E-01	4.6229E-01	4.6601E-01	4.7408E-01	4.9480E-01	5.4269E-01	6.1992E-01	7.1581E-01	8.1354E-01	8.9170E-01
20	4.6292E-01	4.6287E-01	4.6691E-01	4.7525E-01	4.9677E-01	5.4449E-01	6.2020E-01	7.1529E-01	8.1305E-01	8.9153E-01
19	4.6367E-01	4.6361E-01	4.6791E-01	4.7649E-01	4.9862E-01	5.4641E-01	6.2152E-01	7.1593E-01	8.1337E-01	8.9192E-01
18	4.6415E-01	4.6409E-01	4.6861E-01	4.7731E-01	4.9997E-01	5.4805E-01	6.2315E-01	7.1760E-01	8.1516E-01	8.9359E-01
17	4.6426E-01	4.6421E-01	4.6889E-01	4.7760E-01	5.0054E-01	5.4876E-01	6.2417E-01	7.1852E-01	8.1639E-01	8.9490E-01
16	4.6416E-01	4.6412E-01	4.6891E-01	4.7751E-01	5.0057E-01	5.4859E-01	6.2377E-01	7.1765E-01	8.1593E-01	8.9221E-01
15	4.6413E-01	4.6409E-01	4.6882E-01	4.7724E-01	5.0036E-01	5.4827E-01	6.2356E-01	7.1682E-01	8.1571E-01	8.9192E-01
14	4.6421E-01	4.6417E-01	4.6852E-01	4.7695E-01	4.9959E-01	5.4710E-01	6.2297E-01	7.1607E-01	8.1563E-01	8.9155E-01
13	4.6434E-01	4.6430E-01	4.6757E-01	4.7692E-01	4.9733E-01	5.4293E-01	6.2043E-01	7.0037E-01	7.8819E-01	8.6677E-01
12	4.6436E-01	4.6433E-01	4.6513E-01	4.7673E-01	4.9184E-01	5.3252E-01	6.1202E-01	6.8593E-01	7.7522E-01	8.5165E-01
11	4.6366E-01	4.6366E-01	4.6006E-01	4.7369E-01	4.8057E-01	5.1409E-01	5.9399E-01	6.6815E-01	7.5809E-01	8.2524E-01
10	4.6037E-01	4.6049E-01	4.5051E-01	4.6297E-01	4.6143E-01	4.8885E-01	5.6783E-01	6.4777E-01	7.3389E-01	7.8596E-01
9	4.5031E-01	4.5075E-01	4.3318E-01	4.4154E-01	4.3486E-01	4.5979E-01	5.3572E-01	6.2519E-01	7.0205E-01	7.3691E-01
8	4.3015E-01	4.3117E-01	4.0773E-01	4.1089E-01	4.0422E-01	4.2972E-01	4.9906E-01	6.0058E-01	6.6379E-01	6.8165E-01
7	4.0157E-01	4.0329E-01	3.7617E-01	3.7571E-01	3.7371E-01	4.0037E-01	4.6076E-01	5.3777E-01	6.1771E-01	6.2623E-01
6	3.6677E-01	3.6914E-01	3.4258E-01	3.4105E-01	3.4593E-01	3.7276E-01	4.2617E-01	5.4417E-01	5.6746E-01	5.7561E-01
5	3.3007E-01	3.3291E-01	3.1056E-01	3.0878E-01	3.2054E-01	3.4612E-01	3.8997E-01	5.1025E-01	5.1711E-01	5.2972E-01
4	2.9341E-01	2.9650E-01	2.7949E-01	2.7458E-01	2.9392E-01	3.1587E-01	3.5443E-01	4.6812E-01	4.6631E-01	4.8199E-01
3	2.5136E-01	2.5448E-01	2.4141E-01	2.2647E-01	2.5703E-01	2.7262E-01	3.0646E-01	4.0738E-01	4.0400E-01	4.1602E-01
2	1.7802E-01	1.8067E-01	1.6809E-01	1.4420E-01	1.8183E-01	1.8898E-01	2.1206E-01	2.8800E-01	2.8632E-01	2.8400E-01
1	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00

ixi =	20	22	24	26	28	30	32	34	36	38
jbc2	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
51	2.9857E-01	3.0839E-01	3.1712E-01	3.3423E-01	3.4489E-01	3.5445E-01	3.4078E-01	2.2006E-01	1.484E-02	6.8543E-03
50	4.4181E-01	4.5696E-01	4.7099E-01	4.9617E-01	5.1278E-01	5.2884E-01	5.1910E-01	7.2053E-02	2.9590E-02	2.9590E-02
49	5.1606E-01	5.3374E-01	5.5041E-01	5.8024E-01	5.9993E-01	6.1950E-01	6.1413E-01	4.8475E-01	1.4386E-01	8.7239E-02
48	5.6688E-01	5.8594E-01	6.0420E-01	6.3782E-01	6.5945E-01	6.8049E-01	6.7870E-01	5.5610E-01	1.2388E-01	1.4063E-01
47	6.0807E-01	6.2778E-01	6.4721E-01	6.8316E-01	7.0627E-01	7.3020E-01	7.3190E-01	6.1423E-01	2.7641E-01	1.8732E-01
46	6.4822E-01	6.6782E-01	6.8834E-01	7.2360E-01	7.4820E-01	7.7805E-01	7.8393E-01	6.7043E-01	3.3866E-01	2.3068E-01
45	6.9406E-01	7.1250E-01	7.3435E-01	7.6579E-01	7.9226E-01	8.3043E-01	8.4043E-01	7.3464E-01	4.0937E-01	2.7360E-01
44	7.4805E-01	7.6451E-01	7.8792E-01	8.1480E-01	8.4362E-01	8.8658E-01	9.0153E-01	8.1192E-01	4.2066E-01	3.1828E-01
43	8.0779E-01	8.2288E-01	8.4778E-01	8.7208E-01	9.0334E-01	9.4516E-01	9.6474E-01	9.0048E-01	5.2628E-01	3.6654E-01
42	8.6653E-01	8.8324E-01	9.0972E-01	9.3460E-01	9.6740E-01	1.0045E+00	1.0277E+00	9.9025E-01	5.9589E-01	4.1971E-01
41	9.1523E-01	9.3795E-01	9.6736E-01	9.9516E-01	1.0282E+00	1.0614E+00	1.0864E+00	1.0671E+00	6.8936E-01	4.7842E-01
40	9.4570E-01	9.7817E-01	1.0132E+00	1.0436E+00	1.0766E+00	1.1099E+00	1.1337E+00	1.1218E+00	8.0572E-01	5.4227E-01
39	9.5696E-01	9.9654E-01	1.0367E+00	1.0710E+00	1.1063E+00	1.1412E+00	1.1648E+00	1.1575E+00	9.3761E-01	6.0960E-01
38	9.5939E-01	1.0001E+00	1.0412E+00	1.0786E+00	1.1172E+00	1.1551E+00	1.1836E+00	1.1853E+00	1.0535E+00	6.7821E-01
37	9.6296E-01	1.0041E+00	1.0457E+00	1.0844E+00	1.1248E+00	1.1671E+00	1.1994E+00	1.2111E+00	1.1193E+00	7.4455E-01
36	9.6842E-01	1.0101E+00	1.0525E+00	1.0920E+00	1.1329E+00	1.1746E+00	1.2095E+00	1.2260E+00	1.1308E+00	8.0070E-01
35	9.7099E-01	1.0128E+00	1.0548E+00	1.0937E+00	1.1339E+00	1.1755E+00	1.2092E+00	1.2290E+00	1.1266E+00	8.3830E-01
34	9.7133E-01	1.0131E+00	1.0544E+00	1.0926E+00	1.1321E+00	1.1729E+00	1.2077E+00	1.2327E+00	1.1292E+00	8.5456E-01
33	9.7170E-01	1.0136E+00	1.0543E+00	1.0921E+00	1.1313E+00	1.1722E+00	1.2086E+00	1.2380E+00	1.1326E+00	8.5659E-01
32	9.7048E-01	1.0127E+00	1.0532E+00	1.0909E+00	1.1296E+00	1.1698E+00	1.2076E+00	1.2409E+00	1.1343E+00	8.5227E-01
31	9.6732E-01	1.0101E+00	1.0503E+00	1.0870E+00	1.1260E+00	1.1655E+00	1.2054E+00	1.2400E+00	1.1345E+00	8.4482E-01
30	9.6410E-01	1.0075E+00	1.0474E+00	1.0849E+00	1.1225E+00</					

8	6.9487E-01	7.1077E-01	7.2898E-01	7.4564E-01	7.6661E-01	8.1842E-01	8.3081E-01	7.4752E-01	4.1953E-01	2.1866E-01
7	6.4347E-01	6.6336E-01	6.8293E-01	6.9855E-01	7.1872E-01	7.6754E-01	7.7969E-01	6.8887E-01	3.6147E-01	1.8172E-01
6	5.9832E-01	6.2192E-01	6.4176E-01	6.5656E-01	6.7571E-01	7.1781E-01	7.2907E-01	6.3524E-01	3.0379E-01	1.4439E-01
5	5.5569E-01	5.7964E-01	5.9886E-01	6.1272E-01	6.3061E-01	6.6662E-01	6.7591E-01	5.7839E-01	2.4459E-01	1.0459E-01
4	5.0663E-01	5.2756E-01	5.4538E-01	5.5789E-01	5.7406E-01	6.0565E-01	6.1212E-01	5.0971E-01	1.7915E-01	5.9946E-02
3	4.3477E-01	4.5153E-01	4.6683E-01	4.7713E-01	4.9064E-01	5.1675E-01	5.1987E-01	4.1366E-01	1.0360E-01	1.3543E-02
2	2.9507E-01	3.0507E-01	3.1506E-01	3.2100E-01	3.2959E-01	3.4681E-01	3.4569E-01	2.4922E-01	3.4010E-02	1.2889E-02
1	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00

1x1 =	40	42	44	46	48	50	52	54	56	58
1eta										
51	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
50	7.0352E-03	4.3272E-03	4.4874E-03	3.9653E-03	3.3182E-03	2.5516E-03	1.6103E-03	1.4918E-03	9.3575E-04	7.5066E-04
49	3.5135E-02	2.2942E-02	2.2821E-02	2.1054E-02	1.8420E-02	1.6063E-02	1.3835E-02	1.2186E-02	9.9996E-03	8.5432E-03
48	8.4902E-02	6.2613E-02	6.0396E-02	5.6683E-02	5.0542E-02	4.5355E-02	4.0640E-02	3.5982E-02	3.0951E-02	2.7032E-02
47	1.4184E-01	1.1607E-01	1.0994E-01	1.0295E-01	9.2919E-02	8.3868E-02	7.5454E-02	6.7391E-02	5.9492E-02	5.2828E-02
46	1.9407E-01	1.6960E-01	1.5893E-01	1.4775E-01	1.3629E-01	1.2139E-01	1.0905E-01	9.8186E-02	8.7968E-02	7.9173E-02
45	2.3954E-01	2.1744E-01	2.0277E-01	1.8740E-01	1.7069E-01	1.5445E-01	1.3863E-01	1.2570E-01	1.1545E-01	1.0303E-01
44	2.7975E-01	2.5955E-01	2.4170E-01	2.2272E-01	2.0284E-01	1.8374E-01	1.6492E-01	1.5044E-01	1.3621E-01	1.2450E-01
43	3.1641E-01	2.9714E-01	2.7683E-01	2.5507E-01	2.3217E-01	2.0923E-01	1.8923E-01	1.7342E-01	1.5721E-01	1.4435E-01
42	3.5113E-01	3.3166E-01	3.0914E-01	2.8549E-01	2.5985E-01	2.3598E-01	2.1282E-01	1.9555E-01	1.7735E-01	1.6338E-01
41	3.8442E-01	3.6450E-01	3.3982E-01	3.1487E-01	2.8683E-01	2.6097E-01	2.3682E-01	2.1768E-01	1.9748E-01	1.8233E-01
40	4.2046E-01	3.9638E-01	3.6927E-01	3.4401E-01	3.1394E-01	2.8624E-01	2.6212E-01	2.4054E-01	2.1831E-01	2.0191E-01
39	4.6657E-01	4.3120E-01	4.0062E-01	3.7363E-01	3.4158E-01	3.1197E-01	2.8915E-01	2.6478E-01	2.4063E-01	2.2278E-01
38	5.2778E-01	4.7638E-01	4.3995E-01	4.0742E-01	3.7256E-01	3.4020E-01	3.1827E-01	2.9067E-01	2.6496E-01	2.4551E-01
37	6.0089E-01	5.3850E-01	4.9268E-01	4.5189E-01	4.1271E-01	3.7610E-01	3.5161E-01	3.2047E-01	2.9186E-01	2.7026E-01
36	6.7541E-01	6.1515E-01	5.5947E-01	5.1151E-01	4.6751E-01	4.2594E-01	3.9369E-01	3.5883E-01	3.2485E-01	2.9949E-01
35	7.4099E-01	6.8944E-01	6.3260E-01	5.8320E-01	5.3683E-01	4.9211E-01	4.4996E-01	4.1010E-01	3.7037E-01	3.3876E-01
34	7.8919E-01	7.4604E-01	6.9624E-01	6.5226E-01	6.0928E-01	5.6626E-01	5.2699E-01	4.7547E-01	4.3190E-01	3.9316E-01
33	8.1431E-01	7.7654E-01	7.3731E-01	7.0045E-01	6.6510E-01	6.2946E-01	5.9067E-01	5.4644E-01	5.0367E-01	4.6118E-01
32	8.2142E-01	7.8652E-01	7.5497E-01	7.2288E-01	6.9040E-01	6.6442E-01	6.3744E-01	6.0232E-01	5.6866E-01	5.2827E-01
31	8.2056E-01	7.8709E-01	7.5833E-01	7.2878E-01	7.0261E-01	6.7919E-01	6.5612E-01	6.3056E-01	6.0381E-01	5.7404E-01
30	8.1712E-01	7.8486E-01	7.5637E-01	7.2804E-01	7.0239E-01	6.7976E-01	6.5860E-01	6.3799E-01	6.1614E-01	5.9334E-01
29	8.1375E-01	7.8237E-01	7.5343E-01	7.2580E-01	7.0031E-01	6.7758E-01	6.5663E-01	6.3732E-01	6.1690E-01	5.9689E-01
28	8.1075E-01	7.8013E-01	7.5093E-01	7.2367E-01	6.9839E-01	6.7569E-01	6.5478E-01	6.3556E-01	6.1533E-01	5.9576E-01
27	8.0881E-01	7.7862E-01	7.4933E-01	7.2228E-01	6.9725E-01	6.7469E-01	6.5392E-01	6.3465E-01	6.1456E-01	5.9479E-01
26	8.0644E-01	7.7841E-01	7.4927E-01	7.2234E-01	6.9752E-01	6.7511E-01	6.5452E-01	6.3526E-01	6.1531E-01	5.9573E-01
25	8.0936E-01	7.7937E-01	7.5053E-01	7.2373E-01	6.9910E-01	6.7688E-01	6.5651E-01	6.3735E-01	6.1759E-01	5.9811E-01
24	8.1115E-01	7.8113E-01	7.5273E-01	7.2596E-01	7.0129E-01	6.7924E-01	6.5916E-01	6.4022E-01	6.2070E-01	6.0134E-01
23	8.1353E-01	7.8350E-01	7.5545E-01	7.2834E-01	7.0339E-01	6.8160E-01	6.6188E-01	6.4310E-01	6.2390E-01	6.0465E-01
22	8.1605E-01	7.8547E-01	7.5793E-01	7.3035E-01	7.0492E-01	6.8360E-01	6.6442E-01	6.4574E-01	6.2637E-01	6.0646E-01
21	8.1756E-01	7.8619E-01	7.5959E-01	7.3086E-01	7.0531E-01	6.8461E-01	6.6549E-01	6.4538E-01	6.2613E-01	6.0712E-01
20	8.1778E-01	7.8542E-01	7.5902E-01	7.2782E-01	7.0247E-01	6.8063E-01	6.5844E-01	6.3546E-01	6.0660E-01	5.7852E-01
19	8.1648E-01	7.8246E-01	7.5030E-01	7.1509E-01	6.8809E-01	6.5962E-01	6.2870E-01	5.9579E-01	5.6183E-01	5.2894E-01
18	8.1097E-01	7.6834E-01	7.2034E-01	6.8031E-01	6.4522E-01	6.0558E-01	5.6636E-01	5.2924E-01	4.9396E-01	4.6255E-01
17	7.9035E-01	7.2354E-01	6.5737E-01	6.1140E-01	5.6772E-01	5.2375E-01	4.8627E-01	4.5343E-01	4.2485E-01	3.9945E-01
16	7.3003E-01	6.3619E-01	5.6485E-01	5.1921E-01	4.7866E-01	4.4262E-01	4.1395E-01	3.8973E-01	3.7059E-01	3.5033E-01
15	6.2054E-01	5.2759E-01	4.6903E-01	4.3426E-01	4.0503E-01	3.8022E-01	3.6014E-01	3.4387E-01	3.3220E-01	3.1475E-01
14	5.0197E-01	4.3325E-01	3.9368E-01	3.7143E-01	3.5258E-01	3.3633E-01	3.2240E-01	3.1100E-01	3.0316E-01	2.8757E-01
13	4.0775E-01	3.6624E-01	3.4071E-01	3.2767E-01	3.1488E-01	3.0358E-01	2.9358E-01	2.8401E-01	2.7709E-01	2.6949E-01
12	3.4094E-01	3.2072E-01	3.0169E-01	2.9424E-01	2.8362E-01	2.7524E-01	2.6787E-01	2.5975E-01	2.5171E-01	2.4199E-01
11	2.9344E-01	2.8499E-01	2.8815E-01	2.8417E-01	2.5466E-01	2.4848E-01	2.4305E-01	2.3576E-01	2.2651E-01	2.1836E-01
10	2.5700E-01	2.5194E-01	2.3643E-01	2.3485E-01	2.2730E-01	2.2304E-01	2.1903E-01	2.1492E-01	2.1097E-01	2.0316E-01
9	2.2686E-01	2.1983E-01	2.0622E-01	2.0634E-01	2.0073E-01	1.9794E-01	1.9442E-01	1.8978E-01	1.7714E-01	1.8557E-01
8	1.9858E-01	1.8817E-01	1.7662E-01	1.7758E-01	1.7396E-01	1.7219E-01	1.6945E-01	1.6535E-01	1.5207E-01	1.6843E-01
7	1.6845E-01	1.5603E-01	1.4663E-01	1.4780E-01	1.4596E-01	1.4460E-01	1.4195E-01	1.3864E-01	1.2619E-01	1.5099E-01
6	1.3510E-01	1.2219E-01	1.1488E-01	1.1598E-01	1.1539E-01	1.1390E-01	1.1135E-01	1.0845E-01	9.8707E-02	1.3219E-01
5	9.8216E-02	8.4896E-02	7.9764E-02	8.1221E-02	8.1047E-02	7.9145E-02	7.6488E-02	7.4381E-02	6.9489E-02	1.1060E-01
4	5.6864E-02	4.5472E-02	4.2702E-02	4.4534E-02	4.4375E-02	4.2876E-02	4.1244E-02	3.9629E-02	4.0888E-02	8.4116E-02
3	1.8027E-02	1.3852E-02	1.2893E-02	1.4963E-02	1.4970E-02	1.4698E-02	1.4584E-02	1.3619E-02	1.8120E-02	5.1924E-02
2	1.6325E-03	2.1884E-04	1.5458E-05	1.2333E-03	1.4309E-03	1.8499E-03	2.2877E-03	1.9978E-03	4.7820E-03	2.0274E-02
1	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00

1x1 =	60	62	64	66	68	70	72	74	76	78
1eta										
51	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
50	8.1840E-04	1.1972E-03	1.2558E-02	2.7085E-02	4.0362E-02	5.2853E-02	6.2757E-02	7.0280E-02	7.5227E-02	8.0026E-02
49	7.4885E-03	8.0339E-03	3.2478E-02	6.0441E-02	8.0608E-02	9.7488E-02	1.1057E-01	1.2017E-01	1.2654E-01	1.3203E-01
48	2.4356E-02	2.3353E-02	5.5734E-02	8.7335E-02	1.0805E-01	1.2543E-01	1.3889E-01	1.4880E-01	1.5563E-01	1.6096E-01
47	4.8712E-02	4.5263E-02	7.7532E-02	1.0653E-01	1.2630E-01	1.4388E-01	1.5733E-01	1.6760E-01	1.7467E-01	1.7989E-01
46	7.4289E-02	6.8697E-02	1.1232E-01	1.2132E-01	1.4017E-01	1.5782E-01	1.7149E-01	1.8207E-01	1.8945E-01	1.9466E-01
45	9.7625E-02	9.0542E-02	1.2787E-01	1.3599E-01	1.5206E-01	1.6981E-01	1.8371E-01	1.9459E-01	2.0226E-01	2.0754E-01
44	1.1856E-01	1.1048E-01	1.2787E-01	1.4583E-01	1.6320E-01	1.8102E-01	1.9510E-01	2.0626E-01	2.1419E-01	2.1960E-01
43	1.3779E-01	1.2904E-01	1.4237E-01	1.5764E-01	1.7433E-01	1.9213E-01	2.0630E-01	2.1766E-01	2.2578E-01	2.3139E-01
42	1.5605E-01	1.4693E-01	1.5694E-01	1.6990E-01	1.8594E-01	2.0354E-01	2.1769E-01	2.2914E-01	2.3736E-01	2.4320E-01
41	1.7402E-01	1.6472E-01	1.7211E-01	1.8326E-01	1.9836E-01	2.1553E-01	2.2952E-01	2.4093E-01	2.4914E-01	2.5523E-01
40	1.9238E-01	1.8306E-01	1.8834E-01	1.9775E-01	2.1181E-01	2.2832E-01	2.4203E-01	2.5322E-01	2.6130E-01	2.6763E-01
39	2.1173E-01	2.0261E-01	2.0597E-01	2.1366E-01	2.2650E-01	2.4210E-01	2.5541E-01	2.6619E-01	2.7400E-01	2.8054E-01
38	2.3264E-01	2.2406E-01	2.2525E-01	2.3114E-01	2.4259E-01	2.5705E-01	2.6983E-01	2.8002E-01	2.8737E-01	2.9410E-01
37	2.5520E-01	2.4808E-01	2.4630E-01	2.5031E-01	2.6021E-01	2.7329E-01	2.8543E-01	2.9481E-01	3.0155E-01	3.0851E-01
36	2.8135E-01	2.7530E-0								

10	2.0111E-01	1.9811E-01	2.0009E-01	2.0457E-01	2.1064E-01	2.1813E-01	2.2484E-01	2.3293E-01	2.4079E-01	2.4771E-01
9	1.8582E-01	1.8346E-01	1.8613E-01	1.9132E-01	1.9796E-01	2.0583E-01	2.1267E-01	2.2078E-01	2.2866E-01	2.3547E-01
8	1.7127E-01	1.6961E-01	1.7289E-01	1.7868E-01	1.8577E-01	1.9386E-01	2.0077E-01	2.0882E-01	2.1663E-01	2.2331E-01
7	1.5680E-01	1.5593E-01	1.5981E-01	1.6616E-01	1.7363E-01	1.8183E-01	1.8877E-01	1.9671E-01	2.0436E-01	2.1093E-01
6	1.4145E-01	1.4152E-01	1.4607E-01	1.5300E-01	1.6084E-01	1.6911E-01	1.7605E-01	1.8384E-01	1.9129E-01	1.9779E-01
5	1.2377E-01	1.2497E-01	1.3033E-01	1.3796E-01	1.4619E-01	1.5453E-01	1.6147E-01	1.6911E-01	1.7634E-01	1.8281E-01
4	1.0115E-01	1.0376E-01	1.1014E-01	1.1862E-01	1.2736E-01	1.3579E-01	1.4276E-01	1.5026E-01	1.5724E-01	1.6375E-01
3	6.9602E-02	7.3493E-02	8.0689E-02	8.9852E-02	9.8991E-02	1.0742E-01	1.1436E-01	1.2166E-01	1.2832E-01	1.3489E-01
2	3.0503E-02	3.3648E-02	3.9082E-02	4.6185E-02	5.3451E-02	6.0126E-02	6.5697E-02	7.1545E-02	7.6825E-02	8.2454E-02
1	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00

```

ix1 =      80      81
ieta
51 0.0000E+00 0.0000E+00
50 9.1742E-02 9.1742E-02
49 1.4587E-01 1.4587E-01
48 1.7481E-01 1.7481E-01
47 1.9362E-01 1.9362E-01
46 2.0818E-01 2.0818E-01
45 2.2072E-01 2.2072E-01
44 2.3226E-01 2.3226E-01
43 2.4333E-01 2.4333E-01
42 2.5423E-01 2.5423E-01
41 2.6523E-01 2.6523E-01
40 2.7655E-01 2.7655E-01
39 2.8841E-01 2.8841E-01
38 3.0100E-01 3.0100E-01
37 3.1447E-01 3.1447E-01
36 3.2888E-01 3.2888E-01
35 3.4450E-01 3.4450E-01
34 3.6347E-01 3.6347E-01
33 3.8893E-01 3.8893E-01
32 4.2151E-01 4.2151E-01
31 4.5763E-01 4.5763E-01
30 4.9005E-01 4.9005E-01
29 5.1168E-01 5.1168E-01
28 5.2104E-01 5.2104E-01
27 5.2273E-01 5.2273E-01
26 5.2279E-01 5.2279E-01
25 5.2437E-01 5.2437E-01
24 5.2567E-01 5.2567E-01
23 5.2148E-01 5.2148E-01
22 5.0622E-01 5.0622E-01
21 4.7798E-01 4.7798E-01
20 4.4172E-01 4.4172E-01
19 4.0580E-01 4.0580E-01
18 3.7591E-01 3.7591E-01
17 3.5330E-01 3.5330E-01
16 3.3605E-01 3.3605E-01
15 3.2124E-01 3.2124E-01
14 3.0738E-01 3.0738E-01
13 2.9438E-01 2.9438E-01
12 2.8211E-01 2.8211E-01
11 2.7039E-01 2.7039E-01
10 2.5901E-01 2.5901E-01
9 2.4775E-01 2.4775E-01
8 2.3636E-01 2.3636E-01
7 2.2452E-01 2.2452E-01
6 2.1171E-01 2.1171E-01
5 1.9689E-01 1.9689E-01
4 1.7787E-01 1.7787E-01
3 1.4884E-01 1.4884E-01
2 9.4166E-02 9.4166E-02
1 0.0000E+00 0.0000E+00

```

**** Not yet converged.

Note that this calculation has not reached the level of convergence specified in the input ($\Delta Q_{max} \leq 10^{-6}$). However, close examination of several parameters near the end of the calculation indicates that the solution is no longer changing appreciably with time, but oscillates slightly about some mean steady level. This type of result appears to be fairly common, especially for flows with shock waves. The reason is not entirely clear, but may be related to inadequate mesh resolution, discontinuities in metric information, etc. For this particular case, the cause may also be inherent unsteadiness in the flow. The experimental data for this duct show a self-sustained oscillation of the normal shock at Mach numbers greater than about 1.3 (Bogar, Sajben, and Kroutil, 1983).

The three runs for this case required 460.6, 458.2, and 460.5 seconds of CPU time, respectively, for execution. About 161 additional seconds were required in each run to up-dimension and compile the code.

Computed Results

The computed flow field is shown in Figure 9.6 in the form of constant Mach number contours. Contours are plotted at Mach numbers ranging from 0.0 to 1.2 in increments of 0.1.

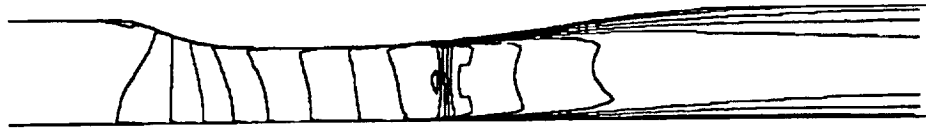


Figure 9.6 - Computed Mach number contours for transonic diffuser flow.

The flow enters the duct at about $M = 0.46$, accelerates to just under $M = 1.3$ slightly downstream of the throat, shocks down to about $M = 0.78$, then decelerates and leaves the duct at about $M = 0.51$. The normal shock in the throat region and the growing boundary layers in the diverging section can be seen clearly. Because this is a shock capturing analysis, the normal shock is smeared in the streamwise direction.

The computed distribution of the static pressure ratio along the top and bottom walls is compared with experimental data (Hsieh, Wardlaw, Collins, and Coakley, 1987) in Figure 9.7. The static pressure ratio is here defined as $p/(p_T)_0$, where $(p_T)_0$ is the inlet core total pressure.

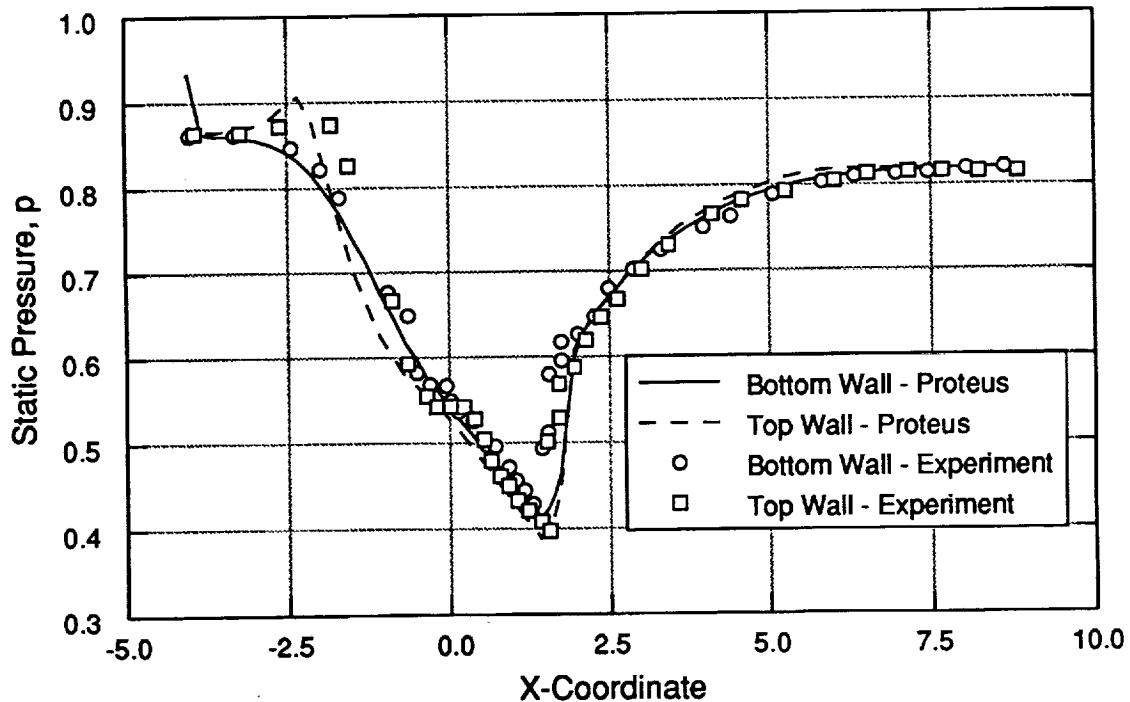


Figure 9.7 - Computed and experimental static pressure distribution for transonic diffuser flow.

The computed results generally agree well with the experimental data, including the jump conditions across the normal shock. The predicted shock position, however, is slightly downstream of the experimentally measured position. The pressure change, of course, is also smeared over a finite distance. There is also some disagreement between analysis and experiment along the top wall near the inlet. This may be due to rapid changes in the wall contour in this region without sufficient mesh resolution.

9.4 TURBULENT FLOW OVER A FLAT PLATE

Problem Description

In this test case, turbulent flow over a flat plate at zero pressure gradient with a freestream Mach number of 0.3 was computed using first the Baldwin-Lomax turbulence model and then the Chien $k-\epsilon$ turbulence model (restarted from the converged Baldwin-Lomax solution). The computational domain ranges from $Re_x = 10,000$ to $Re_x = 10 \times 10^6$. Established experimental data for this flow are available for the purpose of comparison (Klebanoff, 1953; Cole and Hirst, 1968).

Reference Conditions

For convenience, the reference Reynolds number Re , was chosen to be 1×10^6 . To simulate incompressible flows, a freestream (and reference) Mach number M , of 0.3 was selected. Assuming standard sea level values of temperature, kinematic viscosity, and pressure, the reference velocity u , and the reference length L , were calculated to be 334.99 ft/sec and 0.477631 ft, respectively.

Computational Coordinates

The x coordinate for the flat plate under consideration runs from $0.01L$, to $10L$. Grid points are evenly distributed in the x direction. The Cartesian coordinates at the plate (lower boundary) are simply $y = 0$ for all x . For the upper boundary, the y coordinate increases with the distance along the plate to keep approximately the same number of grid points within the boundary layer. To compute the y coordinates, the turbulent boundary layer thickness was first estimated from the power law formula:

$$\delta = \frac{0.38x}{Re_x^{1/5}}$$

Approximately 90 per cent of the grid points are placed inside the turbulent boundary layer, and the rest of the grid points are located in the freestream region. In order to adequately resolve the near wall region, within the boundary layer the grid points in the y direction are clustered near the plate surface using Roberts transformation, as follows:

$$y = \frac{\beta + 1 - (\beta - 1) \left(\frac{\beta + 1}{\beta - 1} \right)^{1-\eta}}{1 + \left(\frac{\beta + 1}{\beta - 1} \right)^{1-\eta}} \delta$$

The packing parameter β was set equal to 1.001. Outside the boundary layer the points are evenly distributed in the y direction.

The coordinate system was generated using a separate program and stored in a Fortran unformatted file to be read by *Proteus*. The Cray runstream used to generate this file is listed below. Note that the Fortran program is included in the JCL as a UNICOS "here document".

```
# QSUB -lM 1.0mW -lT 60
# QSUB -mb -me
# QSUB -o temp.out
# QSUB -eo
# QSUB -r fplate0
# QSUB -s /bin/sh
set -x
ja
touch fcoords
ln fcoords fort.7
cat > coord.f << EOF
  program coord
  c
  c-----Purpose: This program generates the x-y coordinate system for
```

```

c          turbulent flow over a flat plate. The grid increases in
c          height to keep about the same number of points inside
c          the boundary layer. Within the boundary layer, grid
c          points are packed near the plate using Roberts formula.
c
c-----Fortran variable definitions:
c
c          beta      - packing amount parameter in Roberts formula
c          delb      - boundary layer thickness
c          n1, n2    - number of points in x and y directions
c          n3        - number of points in y direction in boundary
c                     layer
c          rer       - reference Reynolds number in Proteus calculation
c          rex       - Reynolds number based on x
c          xmin, xmax - minimum and maximum values of x
c
c-----Output computed:
c
c          x(i,j) - x coordinates at grid points
c          y(i,j) - y coordinates at grid points
c
c          parameter (nxmax=81, nymin=51)
c          dimension x(nxmax,nymin),y(nxmax,nymin)
c          data xmin,xmax /0.01,10.0/
c          data n1,n2 /81,51/
c          data rer /1.e6/
c
c-----Define grid packing parameters
c
c          beta = 1.001
c          n3 = n2 - int(n2/10)
c          deta = 1.0/(n3-1)
c          dx = (xmax-xmin)/float(n1-1)
c
c-----Get x increment, loop in x direction
c
c          do 30 i1 = 1,n1
c             xi = xmin + float(i1-1)*dx
c
c-----Compute y coordinates inside boundary layer
c
c          rex = xi*rer
c          delb = 0.38*xi/rex**0.2
c          do 10 i2 = 1,n3
c             eta = float(i2-1)*deta
c             term1 = ((beta+1.0)/(beta-1.0))*((1.0-eta)
c             top = (beta+1.) - (beta-1.)*term1
c             bottom = term1 + 1.0
c             y(i1,i2) = delb*top/bottom
c             x(i1,i2) = xi
c          10 continue
c          y(i1,1) = 0.0
c
c-----Compute y coordinates outside boundary layer
c
c          do 20 i2 = n3+1,n2
c             y(i1,i2) = y(i1,n3) + (y(i1,n3)-y(i1,n3-1))*(i2-n3)
c             x(i1,i2) = xi
c          20 continue
c          30 continue
c
c-----Write unformatted coordinate system file
c
c          write (7) n1,n2
c          write (7) ((x(i,j),i=1,n1),j=1,n2),
c          $          ((y(i,j),i=1,n1),j=1,n2)
c          close (7)
c          end
c
EOF
cft77 -d pq coord.f
segldr -o coord.ex coord.o

```

```
coord.ex
rm coord.f coord.o coord.ex
ja -cs1t
```

Initial Conditions

The computations are done in two separate major steps: a calculation using the Baldwin-Lomax algebraic turbulence model and a calculation using the Chien $k-\varepsilon$ model. To start the Baldwin-Lomax calculation, the initial velocities u and v were calculated using the Blasius equations for a laminar boundary layer over a flat plate. Non-dimensional values of p were set to 1.0 everywhere in the flow field. To start the Chien calculation, the initial values of u , v , p , and μ , for the turbulent flow field were obtained from the converged Baldwin-Lomax calculation. The initial values of k and ε were obtained using the default KEINIT subroutine.

Boundary Conditions

For both calculations, constant stagnation enthalpy was assumed. Therefore, three boundary conditions were required for the mean flow at each computational boundary. In addition, for the Chien calculation, boundary conditions were required for k and ε at each computational boundary.

For the Baldwin-Lomax calculation, the static pressure was specified at both the upstream and downstream boundaries. The velocity profiles were specified using the Blasius solution at the upstream boundary, and extrapolated at the downstream boundary. At the surface of the plate no-slip conditions were used for the velocity, and the normal pressure gradient was set equal to zero. Freestream conditions were specified at the outer boundary. These boundary conditions are summarized in the following table.

<u>Boundary</u>	<u>Boundary Conditions</u>
$\xi = 0$	$u = u_{\text{Blasius}}, v = v_{\text{Blasius}}, p = 1$
$\xi = 1$	u and v extrapolated, $p = 1$
$\eta = 0$	$u = v = 0, \partial p / \partial \eta = 0$
$\eta = 1$	$u = 1, \partial v / \partial \eta = 0, p = 1$

For the Chien calculation, the boundary conditions for the mean flow were the same as for the Baldwin-Lomax calculation. At the upstream boundary, the gradient of the turbulent kinetic energy k was set equal to zero for the first 1000 time steps. After that time, the value of k was kept constant. The gradient of the turbulent dissipation rate ε was set equal to zero for all time. At the downstream boundary, k and ε were extrapolated. No-slip conditions were used at the surface of the plate, and freestream conditions were used at the outer boundary. The boundary conditions for k and ε are summarized in the following table.

<u>Boundary</u>	<u>Boundary Conditions</u>
$\xi = 0$	$\partial k / \partial \xi = 0$ for $n \leq 1001$ $k = k^{1001}$ for $n > 1001$ $\partial \varepsilon / \partial \xi = 0$
$\xi = 1$	k and ε extrapolated
$\eta = 0$	$k = \varepsilon = 0$
$\eta = 1$	$\partial k / \partial \eta = \partial \varepsilon / \partial \eta = 0$

JCL and Input for Baldwin-Lomax Calculations

The first set of calculations used the Baldwin-Lomax algebraic turbulence model. The Cray UNICOS JCL file for the run is listed below. The contents of the input section of this listing should be compared with the detailed input description in Section 3.0.

```
# QSUB -1M 2.0mW -1T 1800
# QSUB -mb -me
# QSUB -o temp.out
```

```

# QSUB -eo
# QSUB -r fplatel
# QSUB -s /bin/sh
set -x
ja

# Create modifications file

cat > mod1 << EOM
*id newstuff
*d paramsl.19
    parameter (nlp = 81, n2p = 51)
*d paramsl.22
    parameter (neqp = 3)
*purgedk init
*deck init
    subroutine init
c
c-----Purpose:  Set up initial laminar velocity profile using the
c                  Blasius equation.
c
c-----Common blocks
*call paramsl
*call bcl
*call flowl
*call gmtryl
*call iol
*call metricl
*call numl
c
c-----Check for illegal icvars option (must equal 2 for default init)
c
    if (icvars .ne. 2) then
        icvars = 2
        write (nout,10)
10        format (' ***** WARNING - icvars reset to 2 for default init.')
    endif
c
c-----Set initial flow field over the flat plate
c
    do 110 il = 1,n1
        rex = x(il,1)*rer
c
c-----Velocity profile
c
        do 100 i2 = 2,n2
            if (u(il,i2-1) .lt. 1.0) then
                etab = y(il,i2)/x(il,i2)*sqrt(rex)
                call blas (etab,fbn,fbp,fbpp)
                u(il,i2) = fbp
                v(il,i2) = 0.5/sqrt(rex)*(etab*fbp-fbn)
            else
                u(il,i2) = 1.0
                v(il,i2) = v(il,i2-1)
            endif
            w(il,i2) = 0.0
            p(il,i2) = 1.0
            t(il,i2) = 1.0
100        continue
            u(il,1) = 0.0
            v(il,1) = 0.0
            w(il,1) = 0.0
            p(il,1) = 1.0
            t(il,1) = 1.0
110        continue
        return
    end
*dk blas
    subroutine blas (etamax,fn,fp,fpp)
c
c-----Purpose:  Numerically solve the Blasius equation for a laminar

```

```

c          boundary layer profile.
c
dimension y(3),f(3)
etamin = 0.0
m = 0
n = 3
deta = (etamax-etamin)/100.0
y(1) = 0.332213
y(2) = 0.0
y(3) = 0.0
10  if (etamin .le. etamax-deta) then
20      call runge (n,y,f,etamin,deta,m,k)
      if (k .eq. 1) then
          f(1) = -1./2.*y(1)*y(3)
          f(2) = y(1)
          f(3) = y(2)
          go to 20
      else
          go to 10
      endif
  endif
  fpp = y(1)
  fp = y(2)
  fn = y(3)
  return
end
*dk runge
      subroutine runge (n,y,f,x,h,m,k)
c
c-----Purpose: Perform Runge-Kutta calculation by Gills method.
c
dimension y(3),f(3),q(3)
m = m + 1
go to (1,4,5,3,7), m
1  do 2 i = 1,n
    q(i) = 0.0
2  continue
    a = 0.5
    go to 9
3  a = 1.7071067811865475244
4  x = x + 0.5*h
5  do 6 i = 1,n
    y(i) = y(i) + a*(f(i)*h-q(i))
6  q(i) = 2.0*a*h*f(i) + (1.0-3.0*a)*q(i)
    a = 0.292832188134524756
    go to 9
7  do 8 i = 1,n
8  y(i) = y(i) + h*f(i)/6.0 - q(i)/3.0
    m = 0
    k = 2
    go to 10
9  k = 1
10 return
end
EOM

# Set up and link necessary input/output files

touch plotx
touch plotq
touch chist
touch rql
touch rxl
ln fcoords fort.7
ln plotx fort.8
ln plotq fort.9
ln chist fort.10
ln rql fort.12
ln rxl fort.14

# Update code using modifications file

```



```
update -p p2d20.u -i mod1 -c temp -f
```

```
# Compile, load, and run Proteus
```

```
cft77 -d pq -o aggress temp.f
```

```
segldr -o temp.ex temp.o
```

```
temp.ex << EOD
```

```
Flat Plate Turb. Boundary Layer, Baldwin-Lomax Model
```

```
&rstrt
```

```
  irest=1,
```

Write restart files.

```
&end
```

```
&io
```

```
  ivout=1,2,48*0,
```

Print u and v.

```
  iwout2(1)=1,
```

Print surface values.

```
  iprt=100000,
```

Print every 100,000 time levels.

```
  iprt1=4, iprt2=2,
```

Print at every 4th. ξ and 2nd. η index.

```
  iplot=3,
```

Write PLOT3D 3-D plot files.

```
&end
```

```
&gmtry
```

```
  ngeom=10,
```

Get computational coordinates from file.

```
&end
```

```
&flow
```

```
  ihstag=1,
```

Constant stagnation enthalpy.

```
  lr=0.477630871,
```

Set L_r .

```
  machr=0.3, rer=1000000.,
```

Set M_r and Re_r .

```
  gamr=1.4,
```

Constant specific heats.

```
&end
```

```
&bc
```

```
  jbc1(1,1)=41, jbc1(1,2)=41, gbc1(1,1)=1.0, gbc1(1,2)=1.0, p = 1 at  $\xi = 0, 1$ .
```

$u = u_{init}$, u extrapolated at $\xi = 0, 1$.

```
  jbc1(2,1)=10, jbc1(2,2)=14,
```

$v = v_{init}$, v extrapolated at $\xi = 0, 1$.

```
  jbc1(3,1)=20, jbc1(3,2)=24,
```

*$gbc2(1,2)=1.0$, $\partial p / \partial \eta = 0$, $p = 1$
at $\eta = 0, 1$.*

```
  jbc2(1,1)=42, jbc2(1,2)=41,
```

$gbc2(2,2)=1.0$, $u = 0, 1$ at $\eta = 0, 1$.

```
  jbc2(2,1)=11, jbc2(2,2)=11,
```

$v = 0$, $\partial v / \partial \eta = 0$ at $\eta = 0, 1$.

```
  jbc2(3,1)=21, jbc2(3,2)=22,
```

```
&end
```

```
&num
```

```
  n1=81, n2=51,
```

Use an 81×51 mesh.

```
&end
```

```
&time
```

```
  idtmod=1,
```

Recompute $\Delta \tau$ every time step.

```
  ntseq=2,
```

Use 2 time step sequences.

```
  idtau=5, cfl=5.,20.,
```

Spatially varying $\Delta \tau$, CFL = 5, then 20.

```
  ntime=1000,9000,
```

1000 time steps in first sequence, 9000 in second.

```
  ictest=4, eps=3*0.000001,
```

Use $R_{avg} = 10^{-6}$ as convergence criteria.

```
&end
```

```
&turb
```

```
  iturb=1,
```

Turbulent flow, Baldwin-Lomax model.

```
&end
```

```
&ic
```

```
&end
```

```
EOD
```

```
rm -r mod1 temp.f temp.o temp.ex
```

```
ja -cslt
```

JCL and Input for Chien Calculations

The second set of calculations used the Chien two-equation turbulence model. It was run in two steps. The first run was a restart from the converged Baldwin-Lomax calculation. In this run, the gradient of the turbulent kinetic energy k at the upstream boundary was set equal to zero.

The Cray UNICOS JCL file for the first run is listed below. Note that EPS; the required level of convergence, is lower than for the Baldwin-Lomax calculations. This is necessary so that this run can be started from the converged solution of the Baldwin-Lomax calculation. The contents of the input section of this listing should be compared with the detailed input description in Section 3.0.

```
# QSUB -lM 2.0mW -lT 600
# QSUB -mb -me
# QSUB -o temp.out
# QSUB -eo
# QSUB -r fplate2
# QSUB -s /bin/sh
set -x
ja

# Create modifications file

cat > mod1 << EOM
*id newstuff
*d params1.19
    parameter (nlp = 81, n2p = 51)
*d params1.22
    parameter (neqp = 3)
EOM

# Set up and link necessary input/output files

touch plotx
touch plotq
touch chist
touch rq2
touch rx2
ln plotx fort.8
ln plotq fort.9
ln chist fort.10
ln rq1 fort.11
ln rx1 fort.13
ln rq2 fort.12
ln rx2 fort.14

# Update code using modifications file

update -p p2d20.u -i mod1 -c temp -f

# Compile, load, and run Proteus

cft77 -d pq -o aggress temp.f
segldr -o temp.ex temp.o
temp.ex << EOD
Flat Plate Turb. Boundary Layer, Chien Model
&rstrt
    irest=3,
    &end
    &io
        ivout=1,2,106,107,46*0,
        iwout2(1)=1,
        iprt=100000,
        iprt1=4, iprt2=2,
        iplot=3,
    &end
    &gmtry
        ngeom=10,
    &end
    &flow
        ihstag=1,
        lr=0.477630871,
        machr=0.3, rer=1000000.,
        gamr=1.4,
    &end
```

Read restart files, previous run used Baldwin-Lomax.

Print u, v, k, and ε.

Print surface values.

Print every 100,000 time levels.

Print at every 4th. ξ and 2nd. η index.

Write PLOT3D 3-D plot files.

Get computational coordinates from file.

Constant stagnation enthalpy.

Set L.

Set M, and Re.

Constant specific heats.

```

&bc
  jbc1(1,1)= 2, jbc1(1,2)= 4,       $\partial k/\partial \xi = 0$ ,  $k$  extrapolated at  $\xi = 0, 1$ .
  jbc1(2,1)=12, jbc1(2,2)=14,       $\partial \varepsilon/\partial \xi = 0$ ,  $\varepsilon$  extrapolated at  $\xi = 0, 1$ .
  jbc2(1,1)= 1, jbc2(1,2)= 2,       $k = 0$ ,  $\partial k/\partial \eta = 0$  at  $\eta = 0, 1$ .
  jbc2(2,1)=11, jbc2(2,2)=12,       $\varepsilon = 0$ ,  $\partial \varepsilon/\partial \eta = 0$  at  $\eta = 0, 1$ .
  jbc1(1,1)=41, jbc1(1,2)=41, gbc1(1,1)=1.0, gbc1(1,2)=1.0,  $p = 1$  at  $\xi = 0, 1$ .
  jbc1(2,1)=10, jbc1(2,2)=14,       $u = u_{init}$ ,  $u$  extrapolated at  $\xi = 0, 1$ .
  jbc1(3,1)=20, jbc1(3,2)=24,       $v = v_{init}$ ,  $v$  extrapolated at  $\xi = 0, 1$ .
  jbc2(1,1)=42, jbc2(1,2)=41,      gbc2(1,2)=1.0,  $\partial p/\partial \eta = 0$ ,  $p = 1$ 
                                     at  $\eta = 0, 1$ .
                                     gbc2(2,2)=1.0,  $u = 0, 1$  at  $\eta = 0, 1$ .
                                      $v = 0$ ,  $\partial v/\partial \eta = 0$  at  $\eta = 0, 1$ .

  jbc2(2,1)=11, jbc2(2,2)=11,
  jbc2(3,1)=21, jbc2(3,2)=22,
&end
&num
  n1=81, n2=51,
&end
&time
  idtmod=1,
  idtau=5, cfl=2.,
  ntime=1000,
  ictest=4, eps=3*0.0000001,
&end
&turb
  iturb=20,
&end
&ic
&end
EOD

rm -r mod1 temp.f temp.o temp.ex
ja -cslt

```

Use an 81×51 mesh.

Recompute $\Delta \tau$ every time step.
 Spatially varying $\Delta \tau$.
 Limit of 1000 time steps.
 Use $R_{avg} = 10^{-7}$ as convergence criteria.

Turbulent flow, Chien model.

The JCL and input for the second run was similar to that for the first run. The only differences were:

1. The `touch` and `ln` commands for the restart files were changed to:

```

touch rq3
touch rx3
ln rq2 fort.11
ln rx2 fort.13
ln rq3 fort.12
ln rx3 fort.14

```

2. In namelist RSTRT, IREST was set equal to 2, since the previous run used the two-equation turbulence model.
3. In namelist BC, the value of JBCT1(1,1) was changed from 2 to 0. This changed the boundary condition on the turbulent kinetic energy k at the upstream boundary from $\partial k/\partial \xi = 0$ to $k = k_{init}$, where k_{init} is the value at the end of the previous run.
4. In namelist TIME, the CFL number was raised to 10.0, and the maximum number of time steps was increased to 10000.
5. In the QSUB statements, the CPU time limit was raised to 3600.

Standard Proteus Output

The output listing for the third run is shown below. In the flow field printout, only the x-velocity at the last time level, and the boundary parameters, are included.

NASA Lewis Research Center
 Internal Fluid Mechanics Division
 2-d Proteus version 2.0
 January 1992

Flat Plate Turb. Boundary Layer, Chien Model

```

&RSTRT IREST = 2, NRQIN = 11, NRQOUT = 12, NRXIN = 13, NRXOUT = 14, &END
&IO IDEBUG = 20*0, IPLOT = 3, IPLT = 0, IPLTA = 101*0, IPRT = 100000, IPRT1 = 4, IPRT2 = 2, IPRTA = 101*0, IPRT1A = 1,
4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, 64, 68, 72, 76, 80, 84, 88, 92, 96, 100, IPRT2A = 1, 2,
4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48, 50, 52, 54, 56, 58, 60, 62, 64, 66, 68, 70, 72, 74, 76, 78, 80, 82, 84, 86, 88, 90, 92, 94, 96, 98, 100,
IUNITS = 0, IVOUT = 1, 2, 106, 107, 46*0, IWOUT1 = 2*0, IWOUT2 = 1, 0, NGRID = 7, NHIST = 10, NHMAX = 100, NOUT = 6,
NPLOT = 9, NPLOTX = 8, NSCR1 = 20, &END
&GMTRY TAXI = 0, NGEOM = 10, RMAX = 1.0, RMIN = 0.0, THMAX = 90.0, THMIN = 0.0, XMAX = 1.0, XMIN = 0.0, YMAX = 1.0,
YMIN = 0.0, &END
&FLOW GAMR = 1.4, HSTAGR = 0.0, ICVARS = 2, IEULER = 0, IMHAG = 1, ILAMV = 0, ISWIRL = 0, ITHIN = 2*0, KTR = 0.0,
LR = 0.477630871, MACHR = 0.3, MUR = 0.0, PRLR = 0.0, RER = 1000000.0, RG = 1716.0, RHOR = 7.645E-02, TR = 519.0,
UR = 0.0, &END
&BC FBC1 = 510*0.0, FBC2 = 810*0.0, FBCT1 = 204*0.0, FBCT2 = 324*0.0, GBC1 = 1.0, 4*0.0, 1.0, 4*0.0, GBC2 = 5*0.0,
2*1.0, 3*0.0, GBCT1 = 4*0.0, GBCT2 = 4*0.0, GTBC1 = 100*0.0, GTBC2 = 100*0.0, IBC1 = 510*0, IBC2 = 810*0, IBCT1 = 51*0,
51*10, 51*0, 51*10, IBCT2 = 81*0, 81*10, 81*0, 81*10, JBC1 = 41, 10, 20, 2*0, 41, 14, 24, 2*0, JBC2 = 42, 11,
21, 2*0, 41, 11, 22, 2*0, JBCT1 = 0, 12, 4, 14, JBCT2 = 1, 11, 2, 12, JTBC1 = 10*0, JTBC2 = 10*0, KBC1 = 2*0,
KBC2 = 2*0, NTBC = 0, NTBCA = 10*0, &END
&NUM CAVS2E = 5*1.0, CAVS2I = 5*2.0, CAVS4E = 5*1.0, IAV2E = 0, IAV2I = 1, IAV4E = 1, IPACK = 2*0, NI = 81, N2 = 51,
SU = 2*0.0, 2*10000.0, THC = 1.0, 2*0.0, THE = 1.0, 2*0.0, THX = 1.0, 2*0.0, THY = 1.0, 2*0.0,
THZ = 1.0, 2*0.0, &END
&TIME CFL = 10.0, 9*1.0, CFLMAX = 10.0, CFLMIN = 0.5, CHG1 = 4.0E-02, CHG2 = 6.0E-02, DT = 10*1.0E-02, DTF1 = 1.25,
DTF2 = 1.25, DTMAX = 1.0E-02, DTMIN = 1.0E-02, EPS = 3*1.0E-07, 2*1.0E-05, ICHECK = 10, ICTEST = 4, IDTAU = 5,
IDTMOD = 1, NDTVCV = 2, NITAVG = 10, NTIME = 10000, 9*0, NTSEQ = 1, TLIM = 20.0, &END
&TURB APLUS = 26.0, CB = 5.5, CCLAU = 1.68E-02, CCP = 1.6, CKLEB = 0.3, CKMIN = 0.1, CMUR = 9.0E-02, CNA = 2.0,
CNL = 1.7, CONE = 1.35, CTWOR = 1.8, CTHREE = 1.15E-02, CVK = 0.4, CMK = 0.25, ILDAMP = 0, INNER = 1, ITETA = 0,
ITURB = 20, ITXI = 1, IWALL1 = 2*-1, IWALL2 = 2*-1, LWALL1 = 102*0, LWALL2 = 162*0, NTKE = 1, PRT = 0.91, REXT1 = 0.0,
REXT2 = 0.0, TFACT = 1.0, &END

```

Normalizing conditions

```

Length, lr      = 4.7763E-01 ft
Velocity, ur    = 3.3499E+02 ft/sec
Temperature, tr = 5.1900E+02 deg R
Density, rhor   = 7.6450E-02 lbm/ft3
Viscosity, mur  = 1.2232E-05 lbm/ft-sec
Thermal conductivity, ktr = 1.0192E-01 lbm-ft/sec3-deg R
Pressure, rhormur**2 = 8.5789E+03 lbm/ft-sec2
Energy/vol., rhormur**2 = 8.5789E+03 lbm/ft-sec2
Gas constant, ur**2/tr = 2.1622E+02 ft2/sec2-deg R
Specific heat, ur**2/tr = 2.1622E+02 ft2/sec2-deg R
Stagnation enthalpy, ur**2 = 1.1222E+05 ft2/sec2
Time, lr/ur     = 1.4258E-03 sec

```

Reference conditions

```

Reynolds number, rer = 1.0000E+06
Mach number, machr   = 3.0000E-01
Specific heat ratio, gamr = 1.4000E+00
Laminar Prandtl number, prlr = 7.2082E-01
"Reference" Prandtl number, prr = 2.5950E-02
Specific heat at const. press. = 6.0060E+03 ft2/sec2-deg R
Specific heat at const. vol. = 4.2900E+03 ft2/sec2-deg R
Gas constant, rg      = 1.7160E+03 ft2/sec2-deg R
Pressure, pr          = 2.1162E+03 lbf/ft2
Stagnation enthalpy, hstagr = 3.1732E+06 ft2/sec2

```

Boundary condition parameters

```

jbc2(1,2) = 41 gbc2(1,2) = 1.0000E+00
jbc2(2,2) = 11 gbc2(2,2) = 1.0000E+00
jbc2(3,2) = 22 gbc2(3,2) = 0.0000E+00

jbc1(1,1) = 41 gbc1(1,1) = 1.0000E+00
jbc1(2,1) = 10 gbc1(2,1) = 0.0000E+00
jbc1(3,1) = 20 gbc1(3,1) = 0.0000E+00

jbc2(1,1) = 42 gbc2(1,1) = 0.0000E+00
jbc2(2,1) = 11 gbc2(2,1) = 0.0000E+00
jbc2(3,1) = 21 gbc2(3,1) = 0.0000E+00

jbc1(1,2) = 41 gbc1(1,2) = 1.0000E+00
jbc1(2,2) = 14 gbc1(2,2) = 0.0000E+00
jbc1(3,2) = 24 gbc1(3,2) = 0.0000E+00

```

Boundary condition parameters, k-e equations

```

jbc2(1,2) = 2 gbc2(1,2) = 0.0000E+00
jbc2(2,2) = 12 gbc2(2,2) = 0.0000E+00

jbc1(1,2) = 4 gbc1(1,2) = 0.0000E+00
jbc1(2,2) = 14 gbc1(2,2) = 0.0000E+00

jbc2(1,1) = 1 gbc2(1,1) = 0.0000E+00
jbc2(2,1) = 11 gbc2(2,1) = 0.0000E+00

```

x-velocity at time level 15320

ixi =	1	4	8	12	16	20	24	28	32	36
1	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00
2	1.0000E+00	1.0000E+00	1.0000E+00	9.9999E-01	9.9999E-01	9.9999E-01	9.9999E-01	9.9999E-01	9.9999E-01	9.9999E-01
3	1.0000E+00	1.0001E+00	1.0000E+00	9.9999E-01	9.9999E-01	9.9999E-01	9.9999E-01	9.9999E-01	9.9999E-01	9.9999E-01
4	9.9934E-01	9.9986E-01	9.9985E-01	1.0002E+00	1.0007E+00	1.0011E+00	1.0015E+00	1.0017E+00	1.0018E+00	1.0018E+00
5	9.9208E-01	9.9990E-01	1.0005E+00	9.9866E-01	9.9677E-01	9.9475E-01	9.9295E-01	9.9129E-01	9.8977E-01	9.8837E-01
6	9.5957E-01	9.7378E-01	9.7459E-01	9.6955E-01	9.6600E-01	9.6326E-01	9.6101E-01	9.5910E-01	9.5744E-01	9.5599E-01
7	8.7707E-01	9.3320E-01	9.3245E-01	9.2839E-01	9.2569E-01	9.2366E-01	9.2203E-01	9.2067E-01	9.1950E-01	9.1848E-01
8	7.4577E-01	8.9084E-01	8.8902E-01	8.8652E-01	8.8480E-01	8.8369E-01	8.8276E-01	8.8200E-01	8.8136E-01	8.8080E-01
9	5.9560E-01	8.4816E-01	8.4730E-01	8.4643E-01	8.4571E-01	8.4523E-01	8.4489E-01	8.4464E-01	8.4445E-01	8.4430E-01
10	4.5565E-01	8.0482E-01	8.0759E-01	8.0851E-01	8.0864E-01	8.0873E-01	8.0885E-01	8.0901E-01	8.0917E-01	8.0934E-01
11	3.3947E-01	7.6011E-01	7.6922E-01	7.7234E-01	7.7344E-01	7.7410E-01	7.7463E-01	7.7510E-01	7.7554E-01	7.7595E-01
12	2.4687E-01	7.1314E-01	7.3113E-01	7.3707E-01	7.3948E-01	7.4088E-01	7.4188E-01	7.4269E-01	7.4338E-01	7.4400E-01
13	1.8056E-01	6.6284E-01	6.9207E-01	7.0163E-01	7.0583E-01	7.0829E-01	7.0998E-01	7.1126E-01	7.1228E-01	7.1315E-01
14	1.3004E-01	6.0794E-01	6.5072E-01	6.6482E-01	6.7143E-01	6.7537E-01	6.7805E-01	6.8003E-01	6.8157E-01	6.8283E-01
15	9.3106E-02	5.4698E-01	6.0571E-01	6.2541E-01	6.3510E-01	6.4102E-01	6.4506E-01	6.4804E-01	6.5033E-01	6.5218E-01
16	6.6311E-02	4.7857E-01	5.5551E-01	5.8203E-01	5.9558E-01	6.0402E-01	6.0986E-01	6.1418E-01	6.1751E-01	6.2018E-01
17	4.6968E-02	4.0204E-01	4.9849E-01	5.3318E-01	5.5146E-01	5.6306E-01	5.7118E-01	5.7722E-01	5.8192E-01	5.8569E-01
18	3.3052E-02	3.1949E-01	4.3306E-01	4.7714E-01	5.0113E-01	5.1659E-01	5.2753E-01	5.3574E-01	5.4217E-01	5.4737E-01
19	2.3064E-02	2.3842E-01	3.5851E-01	4.1217E-01	4.4268E-01	4.6275E-01	4.7713E-01	4.8802E-01	4.9661E-01	5.0359E-01
20	1.5912E-02	1.6899E-01	2.7756E-01	3.3739E-01	3.7428E-01	3.9942E-01	4.1779E-01	4.3188E-01	4.4309E-01	4.5227E-01
21	1.0793E-02	1.1561E-01	1.9938E-01	2.5590E-01	2.9577E-01	3.2501E-01	3.4726E-01	3.6477E-01	3.7894E-01	3.9068E-01
22	7.1334E-03	7.6587E-02	1.3444E-01	1.7807E-01	2.1318E-01	2.4189E-01	2.6555E-01	2.8523E-01	3.0180E-01	3.1591E-01
23	4.5190E-03	4.8537E-02	8.5609E-02	1.1462E-01	1.3945E-01	1.6140E-01	1.8100E-01	1.9859E-01	2.1440E-01	2.2864E-01

40	4.8801E+00	0.0000E+00	1.0000E+00	2.6941E-03	1.3470E+03	1.0180E+00	-2.3048E-01	-1.2805E+01	-1.7764E-05
41	5.0050E+00	0.0000E+00	1.0000E+00	2.6845E-03	1.3422E+03	1.0180E+00	-2.3344E-01	-1.2969E+01	-1.7992E-05
42	5.1299E+00	0.0000E+00	1.0000E+00	2.6751E-03	1.3376E+03	1.0180E+00	-2.3636E-01	-1.3131E+01	-1.8217E-05
43	5.2548E+00	0.0000E+00	1.0000E+00	2.6661E-03	1.3330E+03	1.0180E+00	-2.3924E-01	-1.3291E+01	-1.8439E-05
44	5.3796E+00	0.0000E+00	1.0000E+00	2.6573E-03	1.3287E+03	1.0180E+00	-2.4209E-01	-1.3449E+01	-1.8658E-05
45	5.5045E+00	0.0000E+00	1.0000E+00	2.6488E-03	1.3244E+03	1.0180E+00	-2.4490E-01	-1.3605E+01	-1.8875E-05
46	5.6294E+00	0.0000E+00	1.0000E+00	2.6404E-03	1.3202E+03	1.0180E+00	-2.4768E-01	-1.3760E+01	-1.9089E-05
47	5.7543E+00	0.0000E+00	1.0000E+00	2.6323E-03	1.3162E+03	1.0180E+00	-2.5042E-01	-1.3912E+01	-1.9301E-05
48	5.8791E+00	0.0000E+00	1.0000E+00	2.6245E-03	1.3122E+03	1.0180E+00	-2.5314E-01	-1.4063E+01	-1.9510E-05
49	6.0040E+00	0.0000E+00	1.0000E+00	2.6168E-03	1.3084E+03	1.0180E+00	-2.5582E-01	-1.4212E+01	-1.9717E-05
50	6.1289E+00	0.0000E+00	1.0000E+00	2.6093E-03	1.3047E+03	1.0180E+00	-2.5847E-01	-1.4360E+01	-1.9921E-05
51	6.2538E+00	0.0000E+00	1.0000E+00	2.6020E-03	1.3010E+03	1.0180E+00	-2.6110E-01	-1.4505E+01	-2.0123E-05
52	6.3786E+00	0.0000E+00	1.0000E+00	2.5949E-03	1.2974E+03	1.0180E+00	-2.6369E-01	-1.4649E+01	-2.0323E-05
53	6.5035E+00	0.0000E+00	1.0000E+00	2.5879E-03	1.2940E+03	1.0180E+00	-2.6626E-01	-1.4792E+01	-2.0521E-05
54	6.6284E+00	0.0000E+00	1.0000E+00	2.5812E-03	1.2906E+03	1.0180E+00	-2.6880E-01	-1.4933E+01	-2.0717E-05
55	6.7533E+00	0.0000E+00	1.0000E+00	2.5745E-03	1.2873E+03	1.0180E+00	-2.7131E-01	-1.5073E+01	-2.0910E-05
56	6.8781E+00	0.0000E+00	1.0000E+00	2.5681E-03	1.2840E+03	1.0180E+00	-2.7380E-01	-1.5211E+01	-2.1102E-05
57	7.0030E+00	0.0000E+00	1.0000E+00	2.5618E-03	1.2809E+03	1.0180E+00	-2.7626E-01	-1.5348E+01	-2.1292E-05
58	7.1279E+00	0.0000E+00	1.0000E+00	2.5556E-03	1.2778E+03	1.0180E+00	-2.7869E-01	-1.5483E+01	-2.1480E-05
59	7.2528E+00	0.0000E+00	1.0000E+00	2.5495E-03	1.2748E+03	1.0180E+00	-2.8110E-01	-1.5617E+01	-2.1665E-05
60	7.3776E+00	0.0000E+00	1.0000E+00	2.5436E-03	1.2718E+03	1.0180E+00	-2.8349E-01	-1.5750E+01	-2.1849E-05
61	7.5025E+00	0.0000E+00	1.0000E+00	2.5378E-03	1.2689E+03	1.0180E+00	-2.8586E-01	-1.5881E+01	-2.2032E-05
62	7.6274E+00	0.0000E+00	1.0000E+00	2.5322E-03	1.2661E+03	1.0180E+00	-2.8820E-01	-1.6011E+01	-2.2212E-05
63	7.7523E+00	0.0000E+00	1.0000E+00	2.5266E-03	1.2633E+03	1.0180E+00	-2.9051E-01	-1.6140E+01	-2.2391E-05
64	7.8771E+00	0.0000E+00	1.0000E+00	2.5212E-03	1.2606E+03	1.0180E+00	-2.9281E-01	-1.6267E+01	-2.2567E-05
65	8.0020E+00	0.0000E+00	1.0000E+00	2.5159E-03	1.2579E+03	1.0180E+00	-2.9508E-01	-1.6393E+01	-2.2743E-05
66	8.1269E+00	0.0000E+00	1.0000E+00	2.5107E-03	1.2553E+03	1.0180E+00	-2.9733E-01	-1.6518E+01	-2.2916E-05
67	8.2518E+00	0.0000E+00	1.0000E+00	2.5056E-03	1.2528E+03	1.0180E+00	-2.9956E-01	-1.6642E+01	-2.3088E-05
68	8.3766E+00	0.0000E+00	1.0000E+00	2.5006E-03	1.2503E+03	1.0180E+00	-3.0177E-01	-1.6765E+01	-2.3258E-05
69	8.5015E+00	0.0000E+00	1.0000E+00	2.4956E-03	1.2478E+03	1.0180E+00	-3.0395E-01	-1.6886E+01	-2.3426E-05
70	8.6264E+00	0.0000E+00	1.0000E+00	2.4908E-03	1.2454E+03	1.0180E+00	-3.0612E-01	-1.7006E+01	-2.3593E-05
71	8.7513E+00	0.0000E+00	1.0000E+00	2.4861E-03	1.2430E+03	1.0180E+00	-3.0826E-01	-1.7126E+01	-2.3758E-05
72	8.8761E+00	0.0000E+00	1.0000E+00	2.4814E-03	1.2407E+03	1.0180E+00	-3.1038E-01	-1.7243E+01	-2.3922E-05
73	9.0010E+00	0.0000E+00	1.0000E+00	2.4769E-03	1.2384E+03	1.0180E+00	-3.1248E-01	-1.7360E+01	-2.4084E-05
74	9.1259E+00	0.0000E+00	1.0000E+00	2.4724E-03	1.2362E+03	1.0180E+00	-3.1456E-01	-1.7476E+01	-2.4244E-05
75	9.2508E+00	0.0000E+00	1.0000E+00	2.4680E-03	1.2340E+03	1.0180E+00	-3.1662E-01	-1.7590E+01	-2.4403E-05
76	9.3756E+00	0.0000E+00	1.0000E+00	2.4637E-03	1.2318E+03	1.0180E+00	-3.1866E-01	-1.7703E+01	-2.4560E-05
77	9.5005E+00	0.0000E+00	1.0000E+00	2.4594E-03	1.2297E+03	1.0180E+00	-3.2068E-01	-1.7816E+01	-2.4716E-05
78	9.6254E+00	0.0000E+00	1.0000E+00	2.4552E-03	1.2276E+03	1.0180E+00	-3.2268E-01	-1.7927E+01	-2.4869E-05
79	9.7503E+00	0.0000E+00	1.0000E+00	2.4510E-03	1.2255E+03	1.0180E+00	-3.2465E-01	-1.8036E+01	-2.5021E-05
80	9.8751E+00	0.0000E+00	1.0000E+00	2.4468E-03	1.2234E+03	1.0180E+00	-3.2657E-01	-1.8143E+01	-2.5169E-05
81	1.0000E+01	0.0000E+00	9.9997E-01	2.4428E-03	1.2214E+03	1.0180E+00	-3.2850E-01	-1.8250E+01	-2.5318E-05

**** Not yet converged.

A converged solution was obtained at time level 4,320 for run 1, the Baldwin-Lomax calculation. The average absolute value of the residual for each of the equations was less than 1×10^{-6} . The calculation using the Chien $k-\epsilon$ model, runs 2 and 3, did not converge to the level specified in the input, but all of the residuals stopped changing after about 7,000 additional iterations. *Proteus* required 1.95×10^{-5} sec/iteration/gridpoint with the Baldwin-Lomax turbulence model and 2.72×10^{-5} sec/iteration/gridpoint with the Chien $k-\epsilon$ turbulence model.

Computed Results

In Figure 9.8, the computed skin friction distribution is compared with the experimental data of Weighart, as reported by Coles (1968). The Chien $k-\epsilon$ turbulence model gave slightly better agreement with the experimental data than the Baldwin-Lomax algebraic model.

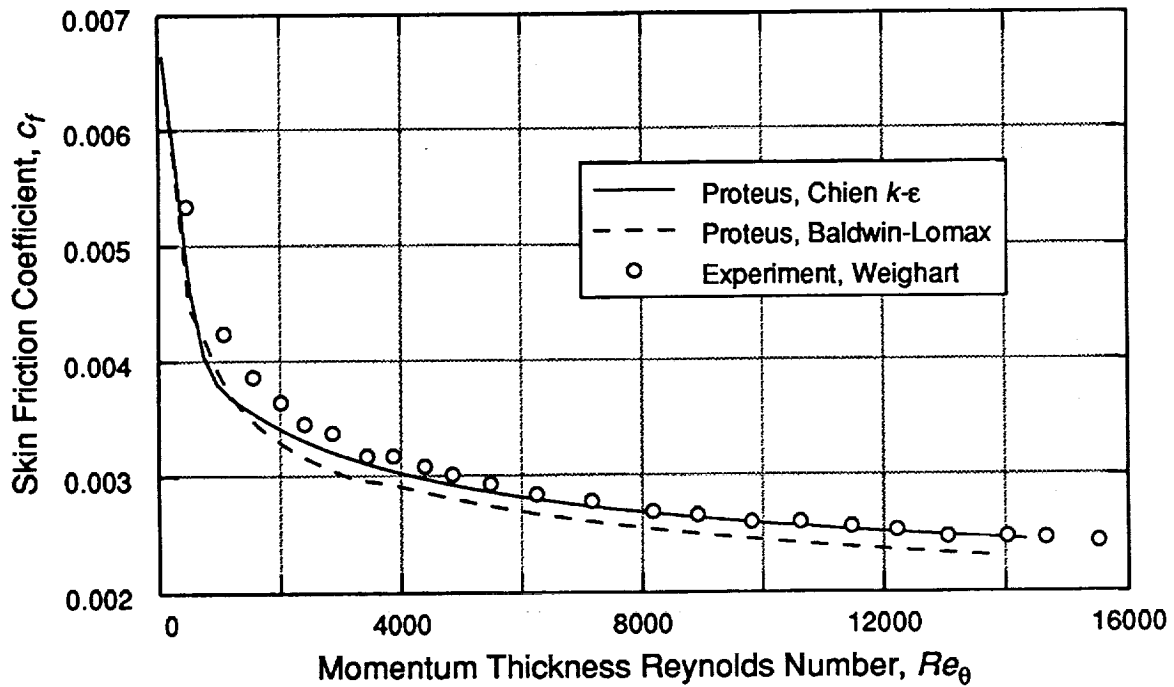


Figure 9.8 - Computed and experimental skin friction coefficient distribution for turbulent boundary layer on a flat plate.

The mean velocity profile for the turbulent boundary layer is shown in Figure 9.9 for $Re_\theta \approx 7700$. The predictions using both the Baldwin-Lomax model and the Chien $k-\epsilon$ model are in excellent agreement with the experimental data of Klebanoff (1953).

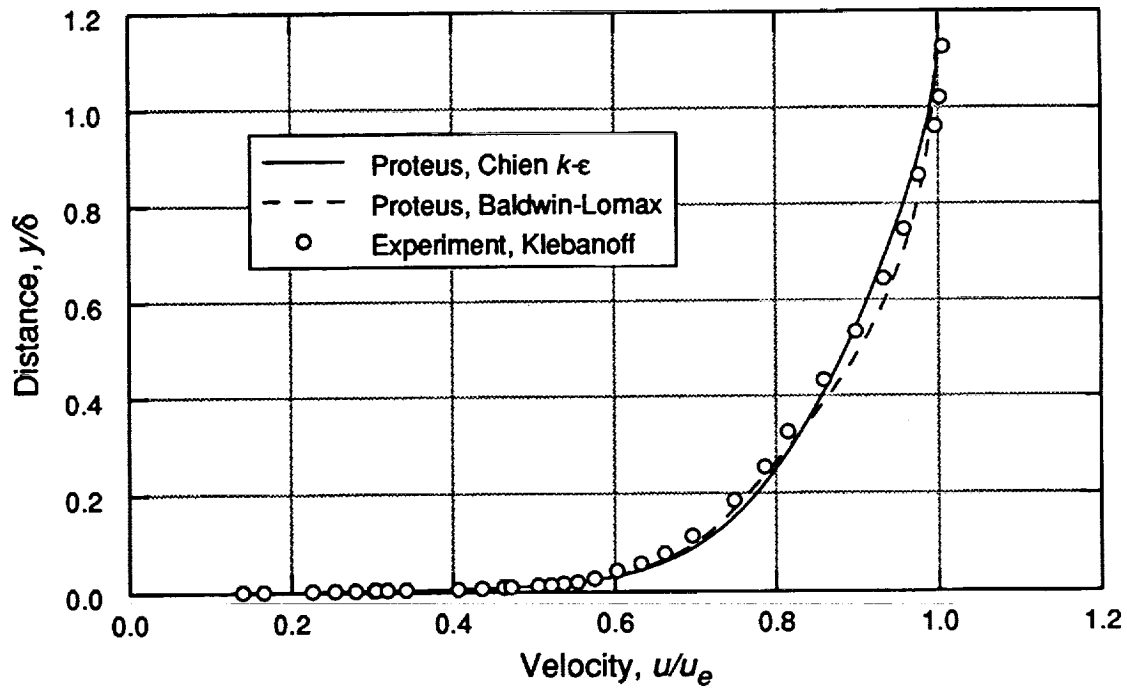


Figure 9.9 - Computed and experimental mean velocity profile for turbulent boundary layer on a flat plate ($Re_\theta \approx 7700$).

The turbulent kinetic energy profile computed using the Chien $k-\epsilon$ turbulence model is compared with the experimental data of Klebanoff (1953) in Figure 9.10. Although the location of the peak in the profile is well predicted, the computed magnitude of the peak was lower than the measured data. This discrepancy is common among many two-equation $k-\epsilon$ models, and the Chien $k-\epsilon$ model is actually one of the better models in this regard (Patel et al, 1985).

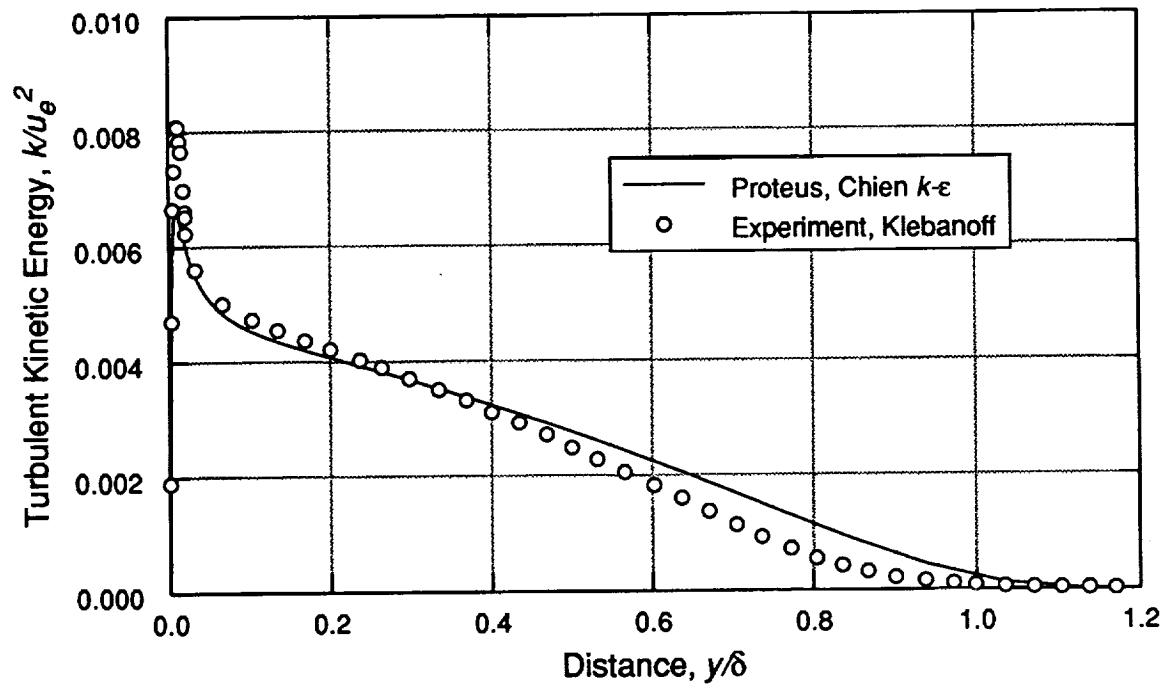


Figure 9.10. Computed and experimental turbulent kinetic energy distribution for turbulent boundary layer on a flat plate ($Re_\theta \approx 7700$).

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13. ABSTRACT (Maximum 200 words) A computer code called <i>Proteus 2D</i> has been developed to solve the two-dimensional planar or axisymmetric, Reynolds-averaged, unsteady compressible Navier-Stokes equations in strong conservation law form. The objective in this effort has been to develop a code for aerospace propulsion applications that is easy to use and easy to modify. Code readability, modularity, and documentation have been emphasized. The governing equations are solved in generalized nonorthogonal body-fitted coordinates, by marching in time using a fully-coupled ADI solution procedure. The boundary conditions are treated implicitly. All terms, including the diffusion terms, are linearized using second-order Taylor series expansions. Turbulence is modeled using either an algebraic or two-equation eddy viscosity model. The thin-layer or Euler equations may also be solved. The energy equation may be eliminated by the assumption of constant total enthalpy. Explicit and implicit artificial viscosity may be used. Several time step options are available for convergence acceleration. The documentation is divided into three volumes. This is the User's Guide, and describes the program's features, the input and output, the procedure for setting up initial conditions, the computer resource requirements, the diagnostic messages that may be generated, the job control language used to run the program, and several test cases.				
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